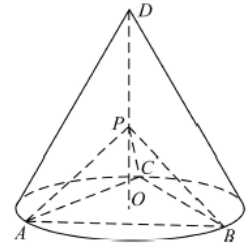


2020-2021

1 2020

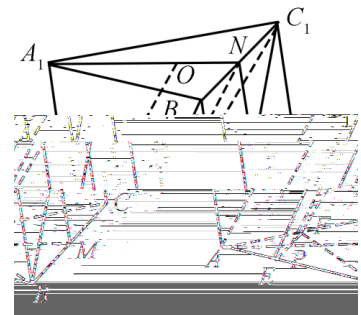
D O
 P DO $APC=90$
 PAB PAC
 $DO=\sqrt{2}$ $\sqrt{3}$ $P-ABC$

ABC



2 2020

$ABC-A_1B_1C_1$
 BB_1C_1C M N BC B_1C_1 P AM
 B_1C_1 P AB E AC F
 $AA_1 \parallel MN$ A_1AMN EB_1C_1F
 O $A_1B_1C_1$ $AO=AB=6$ $AO \parallel$ EB_1C_1F
 $MPN = \frac{1}{3}$ $B-EB_1C_1F$



3 2020

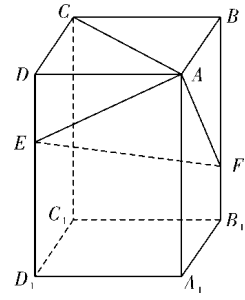
$ABCD - A_1B_1C_1D_1$

$E F$

$DD_1 BB_1$

$2DE = ED_1 \quad BF = 2FB_1$

1 $AB = BC \quad EF \perp AC$ 2 $C_1 \quad AEF$



4 2020

$ABC \quad A_1B_1C_1$

$AB \quad AC \quad B_1C$

ABC

$E F$

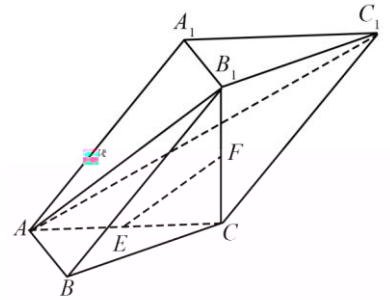
$AC \quad B_1C$

1

$EF \quad AB_1C_1$

2

$AB_1C \quad ABB_1$

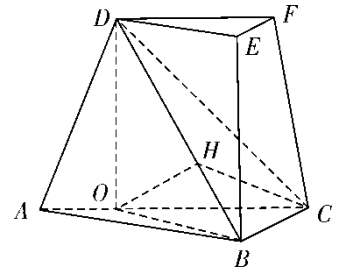


5 2020

$ACB=ACD=45^\circ$ $DC=2BC$
 DBC

$ABC \parallel DEF$ $ACFD$
 $EF \parallel DB$

ABC
 DF

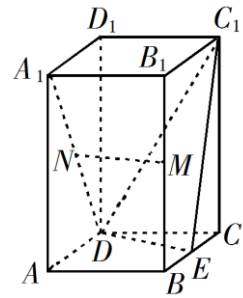


6 2019

E, M, N on BC, BB_1, A_1D respectively.
 1 $MN \parallel C_1DE$ 2 $C \in C_1DE$

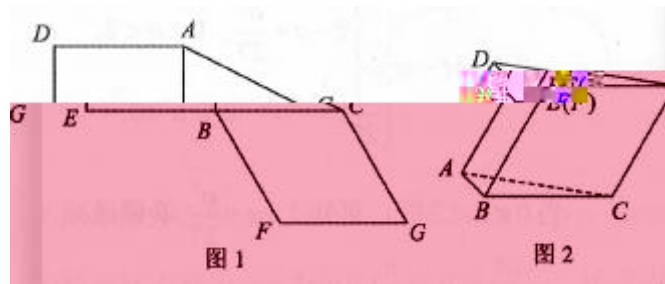
$ABCD \parallel A_1B_1C_1D_1$

$AA_1=4$ $AB=2$ $\angle BAD=60^\circ$



7 2019 $ABCD-A_1B_1C_1D_1$ $ABCD$ E AA_1 BE EC_1
 1 BE EB_1C_1
 2 $AE=A_1E$ $AB=3$ $E-BB_1C_1C$

8 2019 1 $ADEB$ Rt ABC $BFGC$ $AB=1$
 $BE=BF=2$
 $FBC=60^\circ$ AB BC BE BF DG 2
 1 2 A C G D ABC $BCGE$
 2 2 $ACGD$.



2020-2021

1 2020 D O ABC

P DO $APC=90$

1 PAB PAC

2 $DO=\sqrt{2}$ $\sqrt{3}$ $P-ABC$.

1 $PA=PB=PC$

ABC PAC PAB PAC PBC

$APC=90^\circ$ $APB=90^\circ$ $BPC=90^\circ$

PB PA PB PC PB PAC PAB PAC

2 r l $rl=\sqrt{3}$ $l^2-r^2=2$ $r=1$ $l=\sqrt{3}$

$AB=\sqrt{3}$ 1 $PA^2+PB^2=AB^2$ $PA=PB=PC=\frac{\sqrt{6}}{2}$

$P-ABC$ $\frac{1}{3} \times \frac{1}{2} \times PA \times PB \times PC = \frac{1}{3} \times \frac{1}{2} \times (\frac{\sqrt{6}}{2})^3 = \frac{\sqrt{6}}{8}$

2 2020 F $ABC-A_1B_1C_1$ BB_1C_1C M

N BC B_1C_1 P AM B_1C_1 P AB E AC F

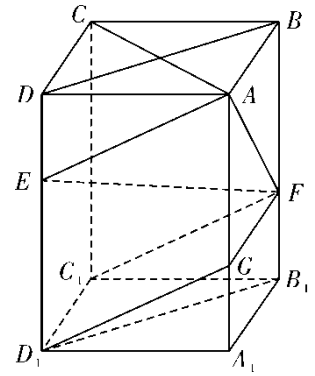
1 $AA_1//MN$ A_1AMN EB_1C_1F 2 O $A_1B_1C_1$ $AO=AB=6$ $AO//$

EB_1C_1F $MPN=\frac{1}{3}$ $B-$

3 2020

$ABCD - A_1B_1C_1D_1$

$E F$



$$DD_1 \parallel BB_1 \quad 2DE = ED_1 \quad BF = 2FB_1$$

$$1 \quad AB = BC \quad EF \perp AC \quad 2 \quad C_1 \quad AEF$$

$$1 \quad BD \parallel B_1D_1 \quad AB = BC \quad ABCD$$

$$AC \perp BD \quad BB_1 \perp ABCD \quad AC \perp BB_1$$

$$AC \perp BB_1D_1D \quad EF \subset BB_1D_1D \quad EF \perp AC$$

$$2 \quad AA_1 \parallel G \quad AG = 2GA_1 \quad GD_1 \parallel FC_1 \parallel FG$$

$$D_1E = \frac{2}{3}DD_1 \quad AG = \frac{2}{3}AA_1 \quad DD_1 \parallel AA_1 \quad ED_1 \parallel AG \quad ED_1GA$$

$$AE \parallel GD_1 \quad B_1F = \frac{1}{3}BB_1 \quad A_1G = \frac{1}{3}AA_1 \quad BB_1 \parallel AA_1 \quad FG \parallel A_1B_1 \quad FG \parallel C_1D_1$$

$$FGD_1C_1 \quad GD_1 \parallel FC_1 \quad AE \parallel FC_1 \quad A, E, F, C_1 \quad C_1 \quad AEF$$

4 2020

$ABC - A_1B_1C_1$ $AB \perp AC$ B_1C

$$ABC \quad E F \quad AC \perp B_1C \quad 1 \quad EF \parallel AB_1C_1$$

$$2 \quad AB_1C \quad ABB_1$$

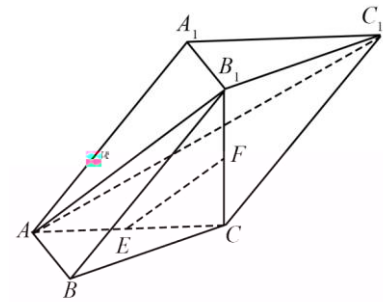
$$1 \quad AC, B_1C \quad EF \parallel AB_1$$

$$EF \not\subset AB_1C_1 \quad AB_1 \subset AB_1C_1 \quad EF \parallel AB_1C_1$$

$$2 \quad B_1C \perp ABC \quad AB \subset ABC \quad B_1C \perp AB$$

$$AB \perp AC, B_1C \subset AB_1C_1, AC \subset AB_1C, B_1C \cap AC = C, AB \perp AB_1C_1 \quad AB \subset$$

$$ABB_1, AB_1C \perp ABB_1$$



5 2020

$ABC - DEF$ $ACFD$ ABC $ACB = ACD = 45^\circ$ DC

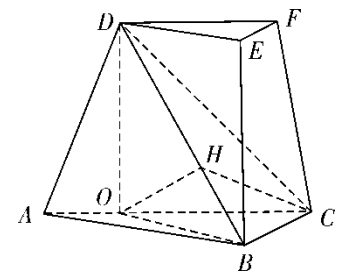
$$= 2BC \quad EF \parallel DB \quad DF \parallel DBC$$

$$D \quad DO \perp AC \quad AC \cap O \quad OB$$

$$\angle ACD = 45^\circ \quad DO \perp AC \quad CD = \sqrt{2}CO$$

$$ACFD \quad ABC \quad DO \perp ABC \quad DO \perp BC$$

$$\angle ACB = 45^\circ \quad BC = \frac{1}{2}CD = \frac{\sqrt{2}}{2}CO \quad BO \perp BC$$



$BC \perp BDO \Rightarrow BC \perp DB.$

$ABC - DEF \Rightarrow BC \perp EF \Rightarrow EF \perp DB.$

$O \Rightarrow OH \perp BD \Rightarrow BD \perp H \Rightarrow CH.$

$ABC - DEF \Rightarrow DF \perp CO \Rightarrow DF \perp DBC \Rightarrow CO \perp DBC.$

$BC \perp BDO \Rightarrow OH \perp BC \Rightarrow OH \perp BCD \Rightarrow \angle OCH = \angle CO \perp DBC.$

$CD = 2\sqrt{2}, DO = OC = 2, BO = BC = \sqrt{2} \Rightarrow BD = \sqrt{6}, OH = \frac{2}{3}\sqrt{3}$

$\sin \angle OCH = \frac{OH}{OC} = \frac{\sqrt{3}}{3} \Rightarrow \frac{DF}{DBC} = \frac{\sqrt{3}}{3}.$

6 2019

$ABCD - A_1B_1C_1D_1 \quad AA_1 = 4$

$AB = 2, \angle BAD = 60^\circ, E, M, N \in BC, BB_1, A_1D$

1 $MN \perp C_1DE$ 2 $C \in C_1DE$

1 $B_1C, ME \perp BB_1, BC$

$ME \perp B_1C \Rightarrow ME = \frac{1}{2}B_1C, N \in A_1D \Rightarrow ND = \frac{1}{2}A_1D.$

$A_1B_1 \perp DC, B_1C \perp A_1D \Rightarrow ME \perp ND$

$MNDE \Rightarrow MN \perp ED, MN \perp C_1DE \Rightarrow MN \perp C_1DE.$

2 $C \in C_1E, H \in DE \perp BC, DE \perp C_1C$

$DE \perp C_1CE \Rightarrow DE \perp CH, CH \perp C_1DE \Rightarrow CH \perp C_1DE$

$CE = 1, C_1C = 4 \Rightarrow C_1E = \sqrt{17}, CH = \frac{4\sqrt{17}}{17} \Rightarrow C \in C_1DE$

$C_1DE \Rightarrow \frac{4\sqrt{17}}{17}.$

7 2019

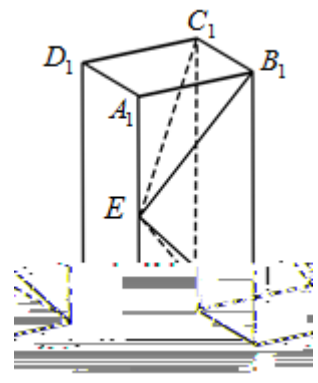
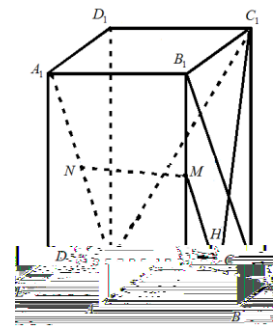
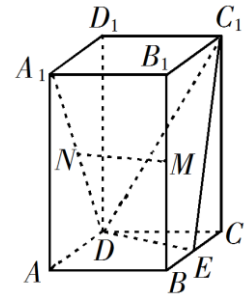
$ABCD - A_1B_1C_1D_1 \quad ABCD$

$E \in AA_1, BE \perp EC_1$

1 $BE \perp EB_1C_1$

2 $AE = A_1E, AB = 3 \Rightarrow E \in BB_1C_1C$

1 $B_1C_1 \perp ABB_1A_1 \Rightarrow BE \perp ABB_1A_1$



$$B_1C_1 \perp BE \quad BE \perp EC_1 \quad BE \quad EB_1C_1$$

$$2 \quad 1 \quad BEB_1=90^\circ$$

$$Rt\triangle ABE \quad Rt\triangle A_1B_1E \quad \angle AEB = \angle A_1EB_1 = 45^\circ$$

$$AE=AB=3 \quad AA_1 = 2AE = 6. \quad EF \perp BB_1 \quad F \quad EF \quad BB_1C_1C \quad EF = AB = 3$$

$$E-BB_1C_1C \quad V = \frac{1}{3} \times 3 \times 6 \times 3 = 18$$

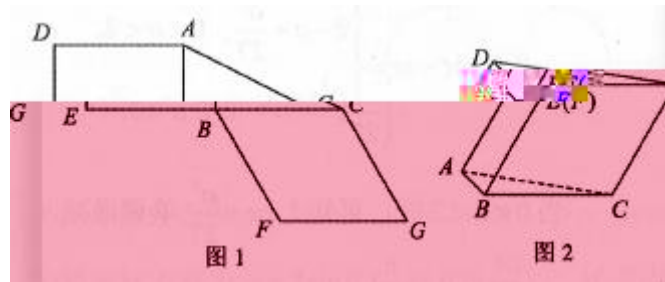
$$8 \quad 2019 \quad 1 \quad ADEB \quad Rt \quad ABC \quad BFGC \quad AB=1$$

$$BE=BF=2$$

$$FBC=60^\circ \quad AB \quad BC \quad BE \quad BF \quad DG \quad 2$$

$$1 \quad 2 \quad A \quad C \quad G \quad D \quad ABC \quad BCGE$$

$$2 \quad 2 \quad ACGD \quad .$$



$$1 \quad AD \parallel BE \quad CG \parallel BE \quad AD \parallel CG \quad AD \quad CG \quad A \quad C \quad G$$

$$D \quad AB \perp BE \quad AB \perp BC \quad AB \perp \quad BCGE$$

$$AB \subset \quad ABC \quad ABC \perp \quad BCGE$$

$$2 \quad CG \quad M \quad EM \quad DM. \quad AB \quad DE \quad AB \perp \quad BCGE \quad DE \perp$$

$$BCGE \quad DE \perp CG.$$

$$BCGE \quad EBC=60^\circ \quad EM \perp CG \quad CG \perp \quad DEM$$

$$DM \perp CG \quad Rt \quad DEM \quad DE=1 \quad EM=\sqrt{3} \quad DM=2 \quad ACGD \quad 4$$

