2020−2021

20200919

8 5 40 .

1. $y=-\frac{\sqrt{3}}{3}x+1$

A $\frac{\pi}{6}$ B $\frac{\pi}{3}$ C $\frac{2\pi}{3}$ D $\frac{5\pi}{6}$

2. $\vec{a}=(x,1,1),\vec{b}=(1,y,1),\vec{c}=(2,-4,2)$ $\vec{a}\perp\vec{c},\vec{b}//\vec{c}$ $|\vec{a}+\vec{b}|=$

A $2\sqrt{2}$ B $\sqrt{10}$ C 3 D 4

3. $ABCD-A_1B_1C_1D_1$ F CD_1 1

A $\frac{1}{2}$ $-\frac{1}{2}$ B $-\frac{1}{2}$ $-\frac{1}{2}$ C $-\frac{1}{2}$ $\frac{1}{2}$ D $\frac{1}{2}$ $\frac{1}{2}$

4. $\vec{p}$ $\{\vec{a},\vec{b},\vec{c}\}$ $(1,2,3)$ $\vec{p}$ $\{\vec{a}+\vec{b},\vec{a}-\vec{b},\vec{c}\}$

A $\left(-\frac{3}{2}\ \frac{1}{2}\ 3\right)$ B $\left(\frac{3}{2}\ -\frac{1}{2}\ 3\right)$ C $\left(\frac{1}{2}\ -\frac{3}{2}\ 3\right)$ D $\left(-\frac{1}{2}\ \frac{3}{2}\ 3\right)$

5. O A,B,C $6\overrightarrow{OP}=\overrightarrow{OA}+2\overrightarrow{OB}+3\overrightarrow{OC}$

A O A B C B P A B C

C O P B C D O P A B C

6. $A(2,-3)$ $B(-3,-2)$ l $kx-y-k+1=0$ l AB l k

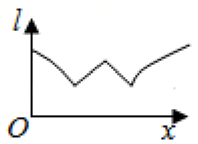
A $k\geq\frac{3}{4}$ $k\leq-4$ B $k\geq\frac{3}{4}$ $k\leq-\frac{1}{4}$

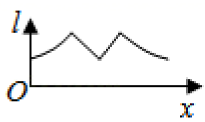
C $-4\leq k\leq\frac{3}{4}$ D $\frac{3}{4}\leq k\leq 4$

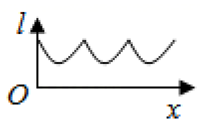
7. MN P 2 $\overrightarrow{PM}\cdot\overrightarrow{PN}$

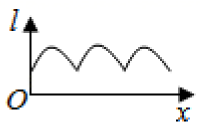
A $[0,4]$ B $[0,2]$ C $[1,4]$ D $[1,2]$

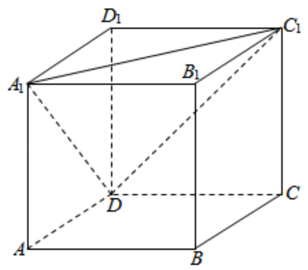
8. $ABCD-A_1B_1C_1D_1$ M B_1 B_1 M A_1DC_1 x $l=MA_1+MC_1+MD$ $l=f(x)$

A 

B 

C 

D 



4 5 20 . .

5 0 3 .

9. $l:(a^2+a+1)x-y+1=0$ $a\in\mathbf{R}$

A $a=-1$ l $x+y=0$ B l $x-y=0$ $a=0$

C l $(0,1)$ D $a=0$ l

10. $\vec{a}=\left(x_1,y_1,z_1\right)$ $\vec{b}=\left(x_2,y_2,z_2\right)$

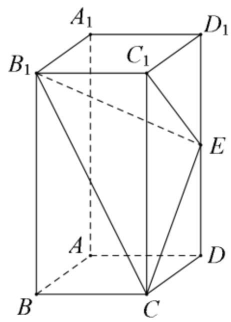
A $\vec{a}\perp\vec{b}$ $x_1x_2+y_1y_2+z_1z_2=0$ B $\vec{a}//\vec{b}$ $\frac{x_1}{x_2}=\frac{y_1}{y_2}=\frac{z_1}{z_2}$

C $\cos<\vec{a}.\vec{b}>=\frac{x_1x_2+y_1y_2+z_1z_2}{\sqrt{x_1^2+y_1^2+z_1^2}\cdot\sqrt{x_2^2+y_2^2+z_2^2}}$ D $x_1=y_1=z_1=1$ $\vec{a}$

11. $l_1:x-y-1=0$ $l_2:(k+1)x+ky+k=0(k\in\mathbf{R})$

A k l_2 90° B k l_1 l_2

C k l_1 l_2 D k l_1 l_2



12. $ABCD-A_1B_1C_1D_1$ 2 4 E DD_1

A $B_1E\perp A_1B$ B $B_1CE//$ A_1BD

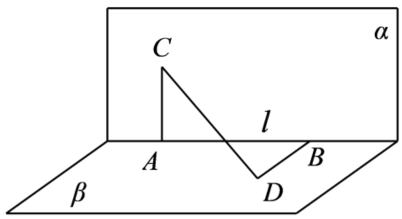
C C_1-B_1CE $\frac{8}{3}$ D $C_1-B_1CD_1$ 24

4 5 20 2 3 .

13. $\vec{a},\vec{b}$ $|\ |$ $|\ |$, $\vec{a}$ $\vec{b}$ $\frac{\pi}{3}$ $\left|\vec{a}+\vec{b}\right|=$________.

14. $l_1:mx+3y=2-m$ $l_2:x+(m+2)y=1$.

$l_1\perp l_2$ $m=$ ________ $l_1//l_2$ $m=$________.



15. $\alpha\perp$ β $\alpha\cap\beta=l$ $A\in l$ $B\in l$

$AC\subset\alpha$ $BD\subset\beta$ $AC\perp l$ $BD\perp l$

$AB=4$ $AC=3$ $BD=12$ $CD=$______________.

16. $\vec{e}_1,\vec{e}_2$, $\vec{e}_1\cdot\vec{e}_2=\frac{1}{2}$, $\vec{b}$ $\left|\vec{b}\right|=2\sqrt{2}$, $\vec{b}\cdot\vec{e}_1=2$, $\vec{b}\cdot\vec{e}_2=\frac{5}{2}$,

$x,y\in\mathbf{R}$, $f\left(x,y\right)=\left|\vec{b}-\left(x\vec{e}_1+y\vec{e}_2\right)\right|$ ________ $x+y=$______.

6 70 17 10

12

17. 10 5

A 12, 13, 14, 15, 16

B 15, 16, 17, 14, a

1 A B

14

$a = 25$

a

18. 12 $\triangle ABC$ A, B, C $5\left(a^2 - b^2\right) = 3bc$ $5\sin C = 8\sin B$

$\angle BAC$ BC D

$\angle BAC$

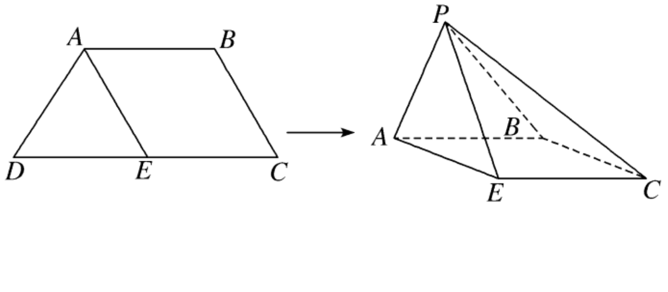
$AC = 5$ AD

19. 12 $ABCD$ $AB // CD$ $AD = BC = AB = 1$ $CD = 2$ E CD

$\triangle ADE$ AE $\triangle APE$

$AE \perp PB$

$P - ABCE$ $A - PE - C$



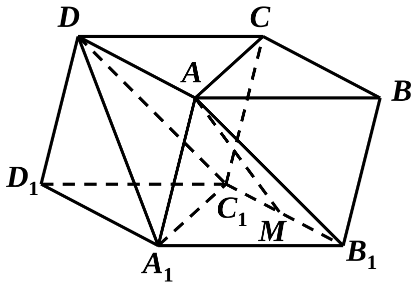
20. 12 $ABCD$ $AD//BC$ $\angle CBD=\angle BDC=\alpha$ $\angle ACD=\beta$.

$\alpha=30^\circ$ $\beta=75^\circ$ $\sqrt{3}AC+\sqrt{2}CD=5$ AC,CD

$\alpha+\beta>90^\circ$ $AB<AD$.

21. 12 $ABCD-A_1B_1C_1D_1$ $ABCD$

$AB_1=A_1B_1=2AA_1=2AC$ $\angle AA_1C_1=\frac{\pi}{3}$ $A_1C_1\perp B_1C_1$

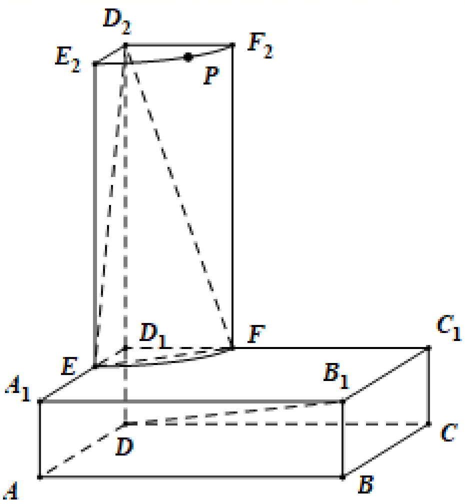


$B_1C_1\perp AA_1$

M B_1C_1 AM DA_1C_1

22. 12 $ABCD-A_1B_1C_1D_1$

4 1 $\sqrt{2}$



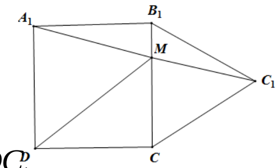
E_2F_2 P B_1 $5\sqrt{2}$ $DB_1\perp$ D_2EF

$D_1D_2=3$ P E_2F_2 $P-A_1C_1-B_1$

2020−2021

8 5 40 .

1−4 DCAB :5−8 BABC



8. AB_1 CB_1 AC $ABCD-A_1B_1C_1D_1$

$AD//B_1C_1$ $AD=B_1C_1$ ADC_1B_1

$DC_1//AB_1$ $DC_1\subset$ A_1DC_1 $AB_1\not\subset$ A_1DC_1 $AB_1//$ A_1DC_1

$CB_1//$ A_1DC_1 $AB_1\cap CB_1=B_1$ $A_1DC_1//$ AB_1C

M B_1AC

$ABCD-A_1B_1C_1D_1$ 1 A_1B_1CD B_1C_1C ,

$0\le x\le\sqrt{2}$ $f(x)=MA_1+MC_1+MD=\sqrt{1+x^2}+\sqrt{1+\left(\sqrt{2}-x\right)^2}+\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2+\left(x-\frac{\sqrt{2}}{2}\right)^2}$

$$f\left(\sqrt{2}-x\right)=\sqrt{1+\left(\sqrt{2}-x\right)^2}+\sqrt{1+x^2}+\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2+\left(\sqrt{2}-x-\frac{\sqrt{2}}{2}\right)^2}=f\left(x\right)$$

$f\left(x\right)$ $\left[0,\sqrt{2}\right]$ $x=\frac{\sqrt{2}}{2}$

$f\left(0\right)=2+\sqrt{3}$ $f\left(\frac{\sqrt{2}}{2}\right)=\sqrt{6}+\frac{\sqrt{2}}{2}$ $f\left(0\right)>f\left(\frac{\sqrt{2}}{2}\right)$

$f\left(x\right)$ $\left[\sqrt{2},2\sqrt{2}\right]$ $x=\frac{3\sqrt{2}}{2}$

$f\left(x\right)$ $\left[2\sqrt{2},3\sqrt{2}\right]$ $x=\frac{5\sqrt{2}}{2}$.

4 5 20 . .

5 0 3 .

9 AC 10 BD 11 AC 12 CD

4 5 20 2 3 .

13 $\sqrt{7}$ 14 $-\frac{3}{2}$ -3 15 13 16 1 3

16. $\because$ $\boldsymbol{e}_1\cdot\boldsymbol{e}_2=|\boldsymbol{e}_1||\boldsymbol{e}_2|\cos<\boldsymbol{e}_1\cdot\boldsymbol{e}_2>=\cos<\boldsymbol{e}_1\cdot\boldsymbol{e}_2>=\frac{1}{2}$ $\therefore$ $<\boldsymbol{e}_1\cdot\boldsymbol{e}_2>=\frac{\pi}{3}$

$\boldsymbol{e}_1=\left(\frac{1}{2},\frac{\sqrt{3}}{2},0\right),\boldsymbol{e}_2=\left(1,0,0\right),\vec{b}=(m,n,t)$

$\because$ $:\vec{b}\cdot\boldsymbol{e}_1=\frac{1}{2}m+\frac{\sqrt{3}}{2}n=2,\vec{b}\cdot\boldsymbol{e}_2=m=\frac{5}{2}$ $\therefore$ $:m=\frac{5}{2},n=\frac{\sqrt{3}}{2}$ $,\boldsymbol{b}=\left(\frac{5}{2},\frac{\sqrt{3}}{2},t\right)$

$\because$ $|\boldsymbol{b}|=2\sqrt{2}$, $t^2=1$ $\because\boldsymbol{b}-(x\boldsymbol{e}_1+y\boldsymbol{e}_2)=\left(\frac{5}{2}-\frac{1}{2}x-y,\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}x,t\right)$

$\therefore$ $f^2\left(x,y\right)=\left|\boldsymbol{b}-\left(x\boldsymbol{e}_1+y\boldsymbol{e}_2\right)\right|^2=\left(x+\frac{y-4}{2}\right)^2+\frac{3}{4}(y-2)^2+t^2$

$=\left|\boldsymbol{b}-\left(x\boldsymbol{e}_1+y\boldsymbol{e}_2\right)\right|^2=\left(x+\frac{y-4}{2}\right)^2+\frac{3}{4}(y-2)^2+1\therefore$ $x=1,y=2$ $f^2\left(x,y\right)$ 1.

6 70 17 10

12

17 5 14 3 $P=\frac{3}{5}$ 3

$a=25$ 1 A B 25

$(12,15)$ $(12,16)$ $(12,17)$ $(12,14)$ $(12,25)$ $(13,15)$ $(13,16)$ $(13,17)$ $(13,14)$ $(13,25)$ $(14,15)$ $(14,16)$ $(14,17)$ $(14,14)$ $(14,25)$ $(15,15)$ $(15,16)$ $(15,17)$ $(15,14)$ $(15,25)$ $(16,15)$ $(16,16)$ $(16,17)$ $(16,14)$ $(16,25)$. 6

$(15,14)$ $(16,15)$ $(16,14)$ 3 7

$P=\frac{3}{25}$. 8

B 14, 15, 16, 17, a

$a=13$ $a=18$ A . 10

18 $5\sin C=8\sin B$ $5c=8b$ $c=\frac{8}{5}b$ 1

$5\left(a^2-b^2\right)=3bc$ $a^2-b^2=\frac{3bc}{5}$. 2

$$\cos\angle BAC=\frac{b^2+c^2-a^2}{2bc}=\frac{-\frac{3}{5}b\times\frac{8}{5}b+\left(\frac{8}{5}b\right)^2}{2b\times\frac{8}{5}b}=\frac{1}{2}$$ 4

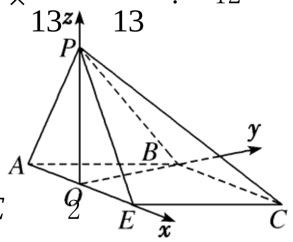
$0<\angle BAC<\pi$ $\angle BAC=\frac{\pi}{3}$. 5

$AC=b=5$ $c=8$ $a^2-25=\frac{3}{5}\times5\times8$ $a^2=49$ $a=7$. 7

$\cos C=\frac{5^2+7^2-8^2}{2\times5\times7}=\frac{1}{7}$ 8 . $\sin C=\sqrt{1-\left(\frac{1}{7}\right)^2}=\frac{4\sqrt{3}}{7}$. 9

$\angle DAC=\frac{\pi}{6}$ $\sin\angle ADC=\sin\left(\frac{\pi}{6}+C\right)=\sin\frac{\pi}{6}\cos C+\cos\frac{\pi}{6}\sin C=\frac{13}{14}$. 10

$\frac{AC}{\sin\angle ADC}=\frac{AD}{\sin C}$ $AD=\frac{AC\cdot\sin C}{\sin\angle ADC}=5\times\frac{4\sqrt{3}}{7}\times\frac{14}{13}=\frac{40\sqrt{3}}{13}$. 12



19 $ABCD$ BD AE O

$AB//CE$ $AB=CE$ $ABCE$

$AE=BC=AD=DE$ $\triangle AED$

$ABCD$ $\angle C=\angle ADE=\frac{\pi}{3}$ $BD\perp BC$ $BD\perp AE$ 2

$OP\perp AE, OB\perp AE$, $OP\bigcap OB=O$ $AE\perp$ POB 4

$PB\subset$ POB $AE\perp PB$ 5

$P-ABCD$ $PAE\perp$ $ABCE$.

$PAE\perp$ $ABCE=AE$ $PO\subset$ PAE $PO\perp AE$ $PO\perp$ $ABCE$.

O x y z

$P(0,0,\frac{\sqrt{3}}{2}),E(\frac{1}{2},0,0),C(1,\frac{\sqrt{3}}{2},0)$ 6

$\overrightarrow{PE}=\left(\frac{1}{2}\ 0\ \ -\frac{\sqrt{3}}{2}\right)$ $\overrightarrow{EC}=(\frac{1}{2},\frac{\sqrt{3}}{2},0)$ 7

PCE $\vec{n}=(x,y,z)$ $\begin{cases}\vec{n}\cdot\overrightarrow{PE}=\frac{1}{2}x-\frac{\sqrt{3}}{2}z=0\\ \vec{n}\cdot\overrightarrow{EC}=\frac{1}{2}x+\frac{\sqrt{3}}{2}y=0\end{cases}$ 9

$x=\sqrt{3}$ $y=-1,z=1$ $\vec{n}=\left(\sqrt{3}\ \ -1\ 1\right)$ PCE 10

PAE $\vec{m}=\left(0\ 1\ 0\right)$ $\cos\langle\vec{m},\vec{n}\rangle=\frac{\vec{m}\cdot\vec{n}}{|\vec{m}||\vec{n}|}=-\frac{\sqrt{5}}{5}$.

$A-PE-C$ $A-PE-C$ $-\frac{\sqrt{5}}{5}$. 12

20 $\angle CBD=\angle BDC=30^\circ$ $\angle ACD=75^\circ$ $\angle ACB=45^\circ$.

AD BC $\angle ADB=\angle CBD=30^\circ$ $\angle DAC=\angle BCA=45^\circ$.

$\angle ADC=60^\circ$. 2

ΔACD $\frac{AC}{\sin\angle ADC}=\frac{CD}{\sin\angle CAD}$ $\frac{AC}{\sin 60^\circ}=\frac{CD}{\sin 45^\circ}$

$AC=\frac{\sqrt{6}}{2}CD$. 4

$\sqrt{3}AC+\sqrt{2}CD=5$ $AC=\sqrt{3}$ $CD=\sqrt{2}$. 6

ΔACB $AB=\sqrt{AC^2+BC^2-2AC\times BC\cos\angle ACB}$. 7

ΔACD

$AD=\sqrt{AC^2+DC^2-2AC\times DC\cos\angle ACD}=\sqrt{AC^2+BC^2-2AC\times BC\cos\angle ACD}$. 9

$\alpha+\beta>90^\circ$ $\angle ACB=180^\circ-2\alpha-\beta$

$\angle ACB-\angle ACD=\left(180^\circ-2\alpha-\beta\right)-\beta=180^\circ-2\left(\alpha+\beta\right)<0$ $\angle ACB<\angle ACD$ 11

$0^\circ<\angle ACB<180^\circ$ $0^\circ<\angle ACD<180^\circ$ $\cos\angle ACB>\cos\angle ACD$

$AB<AD$. 12

21 $A_1C_1//AC,\ A_1C_1=AC$ AA_1C_1C

$AA_1=AC$ AA_1C_1C $\angle AA_1C=\frac{\pi}{3}$

AA_1C 1

AC_1 $AA_1=2a$ $AC_1=2a,\ AB_1=4a$ $B_1C_1=\sqrt{A_1B_1^2-A_1C_1^2}=2\sqrt{3}a$

$AB_1^2=AC_1^2+B_1C_1^2$ $B_1C_1\perp AC_1$ 3

$A_1C_1\perp B_1C_1,\ A_1C_1\cap AC_1=C_1$ $B_1C_1\perp$ ACC_1A_1 4

$AA_1\subset$ ACC_1A_1 $B_1C_1\perp AA_1$ 5

A_1B_1 E A_1C_1 O OE $OE//B_1C_1$ $OE\perp A_1C_1$

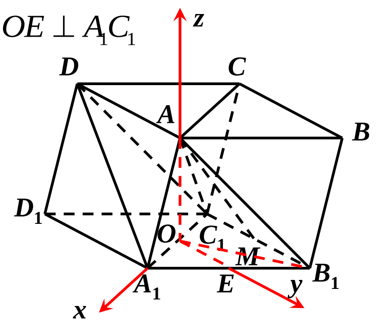
$AO\perp A_1C_1$ $AO\perp B_1C_1$ $B_1C_1\cap A_1C_1=C_1$

$AO\perp$ $A_1B_1C_1$

$AO=\sqrt{3}a$ $a=1$ O

$\overrightarrow{OA_1},\overrightarrow{OE},\overrightarrow{OA}$ x,y,z

$O-xyz$



$A_1(1,0,0), C_1(-1,0,0), B_1(-1,2\sqrt{3},0), A(0,0,\sqrt{3}), C(-2,0,\sqrt{3}), M(-1,\sqrt{3},0)$

$\overrightarrow{C_1A_1}=(2,0,0), \overrightarrow{AM}=(-1,\sqrt{3},-\sqrt{3})$ 8

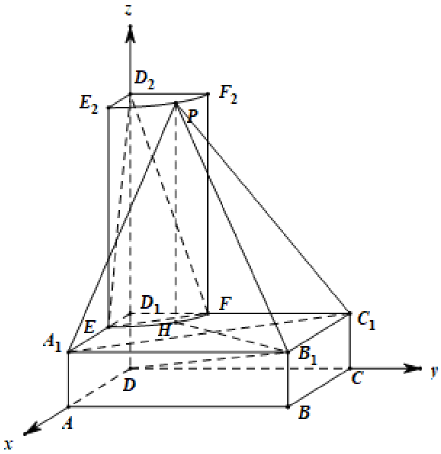
$\overrightarrow{DC}=\overrightarrow{A_1B_1}=(-2,2\sqrt{3},0), \overrightarrow{C_1C}=(-1,0,\sqrt{3})$ $\overrightarrow{C_1D}=\overrightarrow{C_1C}-\overrightarrow{DC}=(1,-2\sqrt{3},\sqrt{3})$ 9

DA_1C_1 $\vec{n}=(x,y,z)$ $\begin{cases}\vec{n}\cdot\overrightarrow{C_1D}=x-2\sqrt{3}y+\sqrt{3}z=0\\ \vec{n}\cdot\overrightarrow{C_1A_1}=2x=0\end{cases}$

$y=1$ $\vec{n}=(0,1,2)$ 11

AM DA_1C_1 θ

$\sin\theta=\left|\cos\langle\overrightarrow{AM},\vec{n}\rangle\right|=\left|\frac{0+\sqrt{3}-2\sqrt{3}}{\sqrt{5}\times\sqrt{7}}\right|=\frac{\sqrt{105}}{35}$ 12



22 $PH\perp$ $A_1B_1C_1D_1$ H H EF

$PB_1=\sqrt{PH^2+HB_1^2}$ HB_1 PB_1

HB_1 $4\sqrt{2}-\sqrt{2}=3\sqrt{2}$

$PH=\sqrt{PB_1^2-HB_1^2}=4\sqrt{2}$, 2

D $\overrightarrow{DA},\overrightarrow{DC},\overrightarrow{DD_2}$ x,y,z

$D(0,0,0), D_2(0,0,1+4\sqrt{2}), E(\sqrt{2},0,1), F(0,\sqrt{2},1), B_1(4,4,1)$

$\overrightarrow{DB_1}=(4,4,1), \overrightarrow{EF}=(-\sqrt{2},\sqrt{2},0), \overrightarrow{ED_2}=(-\sqrt{2},0,4\sqrt{2})$, 3

$\overrightarrow{DB_1}\cdot\overrightarrow{EF}=-4\sqrt{2}+4\sqrt{2}+0=0, \overrightarrow{DB_1}\cdot\overrightarrow{ED_2}=-4\sqrt{2}+0+4\sqrt{2}=0$

$DB_1\perp EF, DB_1\perp ED_2$, $EF\subset$ D_2EF $ED_2\subset$ D_2EF $ED_2\cap EF=E$,

$DB_1\perp$ D_2EF 5

$D_1D_2=3$ $A_1(4,0,1), C_1(0,4,1), B_1(4,4,1)$,

$P(a,b,4)$ $a^2+b^2=2, a\ge 0, b\ge 0$ $a=\sqrt{2}\cos\theta, b=\sqrt{2}\sin\theta, \theta\in[0,\frac{\pi}{2}]$

$a+b=2\sin(\theta+\frac{\pi}{4})\in[\sqrt{2},2]$ $A_1C_1=(-4,4,0), A_1P=(a-4,b,3)$ 6

PA_1C_1 $\vec{n}=(x_1,y_1,z_1)$ $\begin{cases}\vec{n}\cdot\overline{A_1C_1}=-4x_1+4y_1=0\\ \vec{n}\cdot\overline{A_1P}=(a-4)x_1+by_1+3z_1=0\end{cases}$

$x_1=1$ $\vec{n}=(1,1,\frac{4-a-b}{3})$ 8

$A_1B_1C_1$ $\vec{m}=(0,0,1)$

$P-A_1C_1-B_1$ θ θ

$\cos\theta=-\left|\cos\langle\vec{m},\vec{n}\rangle\right|=-\frac{|\vec{m}\cdot\vec{n}|}{|\vec{m}||\vec{n}|}=\frac{\frac{a+b-4}{3}}{\sqrt{2+(\frac{a+b-4}{3})^2}}$ 10

$\sin\theta=\sqrt{1-\cos^2\theta}=\frac{\sqrt{2}}{\sqrt{2+(\frac{a+b-4}{3})^2}}$ $\tan\theta=\frac{3\sqrt{2}}{a+b-4}\in[-\frac{3\sqrt{2}}{2},-\frac{6\sqrt{2}+3}{7}]$

$P-A_1C_1-B_1$ $[-\frac{3\sqrt{2}}{2},-\frac{6\sqrt{2}+3}{7}]$. 12