

2020

1

2

9 $f(x) = \sin(2x - \frac{\pi}{3})$ $x = \frac{k\pi}{2} - \frac{\pi}{12}$ $k \in \mathbf{Z}$

$m, n \in (\frac{1}{2}, 2)$ $\frac{m}{2} - \frac{n}{12} = \frac{3}{2}$ $\frac{3(m-n)}{2} = \frac{9}{4}$

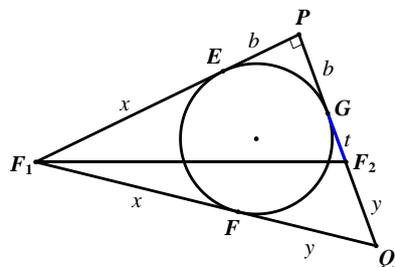
$f(\frac{3m-3n}{2}) = \sin(2(\frac{3}{4} - \frac{9}{2})) = \frac{1}{3}$ $\sin(\frac{1}{2} - \frac{3}{3} - 9) = \sin(\frac{11}{6}) = \frac{1}{2}$

10 A $f(x) = f(x) - 2\cos x$ $g(x) = f(x) - \cos x$
 $x=0$ $f'(x) = \sin x = 0$ $g'(x) = 0$ $g(x) \in [0, \frac{\pi}{2})$

$f(x) = f(\frac{\pi}{2}) = \cos x$ $g(x) = g(\frac{\pi}{2}) = [\frac{\pi}{2}, \frac{\pi}{2})$ C.

11 $b^2 - x^2 = b^2 - t^2 - 2a^2$ $x = 2a$
 $x^2 - y^2 = y^2 - t^2 - 2a^2$ $t = 0$

Rt $\triangle PF_1F_2$ $4c^2 = 2a^2 - b^2$ $b^2 = b^2 - 2a^2$ $e = \sqrt{5}$ B.

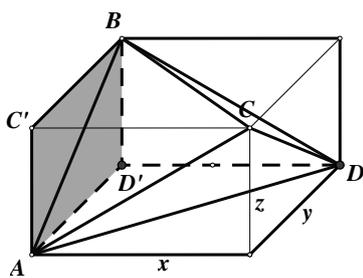


$\angle F_1PQ = 90^\circ$ $r = \frac{PF_1 \cdot PQ \cdot F_1Q}{2} = \frac{PF_1 \cdot PF_2 \cdot QF_2 \cdot QF_1}{2} = \frac{PF_1 \cdot PF_2 \cdot 2a}{2}$

$PF_1 \cdot PF_2 = 2b^2 - 2a^2$ $PF_1 \cdot PF_2 = 2a^2$ $PF_1 = b - 2a, PF_2 = b$

Rt $\triangle PF_1F_2$ $4c^2 = 2a^2 - b^2$ $b^2 = b^2 - 2a^2$ $e = \sqrt{5}$ B. $r = \sqrt{b^2 - c^2} = a$

12



$AD'BC' = yz$

$y^2 + z^2 = 6$

$z^2 + x^2 = a^2$ $S = yz = \frac{y^2 + z^2}{2} = 3$ $y = z = \sqrt{3}$ $a = b$

$x^2 + y^2 = b^2$

$a = 2b = 6$ $a = b = 2$ $x = 1$ $4R^2 = l^2 = x^2 + y^2 + z^2 = 7$ $R = \frac{\sqrt{7}}{2}$

A.

13 3.2 14 2^{25} 15 16 16 $2\sqrt{2}$ 3

13 $\bar{x} = 10, \bar{y} = 8$ $\bar{y} = b\bar{x} + a$ $b = 3.2$

14 $a_n a_{n-1} = 2^n$ $a_{n-1} a_{n-2} = 2^{n-1}$ $\frac{a_{n-2}}{a_n} = 2$ $a_2, a_4, a_6, \dots, a_{2k}$ $a_2 = 2$
 $a_1 = 1, a_2 = 2^1, a_3 = 2, a_4 = 2^2, a_5 = 2^2, a_6 = 2^3$ $a_{2k} = 2^k$ $a_{50} = 2^{25}$.

15 $\overline{BA} \overline{BD} \overline{DC} \overline{DB} \overline{BA} \overline{CD} \overline{BD} \left| \overline{BD} \right| \overline{BA} \overline{CD} \overline{BD}$

$= |\overline{BD}|^2 = 16$.

16 $BF = C$ A'
 $x_F = x_E$ A', A x

$BF = x + ty = \frac{p}{2}$ $x + ty = \frac{p}{2}$
 $y = 2px$
 $y^2 - 2pty + p^2 = 0$ $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = y_2 \begin{pmatrix} 2pt \\ p^2 \end{pmatrix}$ $D(0, 1)$ $t = \frac{p}{2}$

$y_2 = y_1 + y_2 y_1 = \frac{1}{y_1} + \frac{1}{y_2} = 1$

$(y_2 - 2y_1) \left(\frac{1}{y_1} - \frac{1}{y_2} \right) = 3$ $\frac{2y_1 - y_2}{y_2 - y_1} = 3$ $2\sqrt{2}$ $2\sqrt{2} = 3$.

70

17 12

1 ABC $2c = a + b \frac{\sin B}{\sin C} = b^2 + c^2 - a^2$

A 2

10

$$P(A) = 1 - P(\bar{A}) = 1 - \frac{C_{10}^1 C_{10}^1}{C_{20}^2} = \frac{9}{19} \dots\dots\dots 2$$

0.1. 3

$Y \sim B(10, 0.1)$, 4

p

$$P(Y = 1) = 1 - P(Y = 0) - P(Y = 1) = 1 - 0.9^{10} - C_{10}^1 0.9^9 0.1 = 0.2639$$

$X \sim B(6, 0.2639)$, 6

$$EX = 6 \cdot 0.2639 = 1.5834 \dots\dots\dots 7$$

1, 2 $m(m-2)$

$$P(X = k) = (1-p)^{k-1} p, k \in \{1, 2, 3, \dots, m-1\} \quad P(X = m) = (1-p)^{m-1} \dots\dots\dots 9$$

$$E[(1-p)^X] = (1-p) + p(1-p)^2 + p^2(1-p)^3 + p^3(1-p)^4 + \dots + p^{m-1}(1-p)^m + p^m(1-p)^{m-1} \dots\dots\dots 10$$

$(1-p)E$

$$1 - (1-p) + p(1-p)^2 - p^2(1-p)^3 + p^3(1-p)^4 - \dots + p^{m-1}(1-p)^m - p^m(1-p)^{m-1}$$

$$pE = p(1-p) + p^2(1-p)^2 + \dots + p^m(1-p)^m - m(1-p)^m$$

$$pE = \frac{p[1 - (1-p)^{m-1}]}{1 - (1-p)} - m(1-p)^m = 1 - (1-p)^m$$

$$E = \frac{1 - (1-p)^m}{p} \dots\dots\dots \frac{1 - (1-p)^m}{p}$$

21 12

$$f'(x) = e^{x-1} \left(\frac{a}{x} - 2x \right) \quad f(x) = 0 \quad f(1) = x-1 \quad f(x)$$

$$f'(1) = 0 \quad a = 3$$

$$f'(x) = e^{x-1} \left(\frac{3}{x} - 2x \right) = 0,$$

$$f(x) = (0, 1) \quad (1, \dots)$$

$$f(x) = f(1) = 0 \dots$$

$$a = 3.$$

4

$$\begin{array}{ccccccc}
 2 & x & 1 & f & 1 & 0 & h & 1 & 0 & x & 1 & h & x \\
 & 0 & x & 1 & & a & 0 & h & x & g & x & 0 & h & x \\
 & & & & & a & 0 & g & x & 0 & h & x & & f & x \\
 a & 0 & f' & x & e^{x-1} & \frac{a}{x} & 2x & 0 & f & x & (0,1) \\
 f & x & f & 1 & 0 & f & x & (0,1) & & & & & & & .
 \end{array}$$

$$\begin{array}{ccccccc}
 x & 1 & & a & 0 & h & x & g & x & 0 & h & x \\
 & & & a & 0 & g & x & 0 & h & x & & f & x
 \end{array}$$

$$\begin{array}{ccccccc}
 a & 0 & f' & x & e^{x-1} & \frac{a}{x} & 2x & 1, & f' & 1 & 3 & a \\
 0 & a & 3 & f' & 1 & 0 & f' & x & f' & 1 & 0 \\
 & & & f & x & 1, & f & x & f & 1 & 0 \\
 & & & f & x & 1, & & & & & & & & .
 \end{array}$$

$$\begin{array}{ccccccc}
 a & 3 & f' & 1 & 0 & f' & a & e^{a-1} & \frac{a}{a} & 2a & e^{a-1} & 2a & 1 & 0 \\
 & & & x_0 & 1, a & & & & & & & & & \\
 & & & x & 1, x_0 & f' & x & 0 & f & x & & x & x_0, \\
 f' & x & 0 & f & x & & & & f & x_0 & f & 1 & 0 \\
 f & a & e^{a-1} & a \ln a & a^2 & 2 & e^2 & a^2 & a^2 & 2 & 0 \\
 f & x & 1, & & & & & & & & & & .
 \end{array}$$

$$\begin{array}{cccc}
 a & , 0 \cup (0, 3] & h & x & 1 \\
 a & 3, & h & x & 2.
 \end{array}$$

() 10 22 23

22 4 4 10

$$\begin{array}{ccc}
 x' & x & \\
 y' & \frac{\sqrt{3}}{2}y & C_2 \quad \begin{array}{l} x' = 2\cos t \\ y' = \sqrt{3}\sin t \end{array} t
 \end{array}$$

$$\frac{x'^2}{4} + \frac{y'^2}{3} = 1$$

$$x' = \cos t, y' = \sin t \quad \begin{array}{l} \cos^2 t = \frac{12}{3\cos^2 t} \\ \sin^2 t = \frac{12}{4\sin^2 t} \end{array}$$

$$C_2 \quad \begin{array}{l} \cos^2 t = \frac{12}{3\cos^2 t} \\ \sin^2 t = \frac{12}{4\sin^2 t} \end{array}$$

3

$$\begin{aligned}
& P_1, Q_2, \frac{\sqrt{2}}{2} \\
& \frac{OP}{OQ} = \frac{|PQ|^2 + |OP|^2 - |OQ|^2}{2|OP||OQ|} \\
& \left(\frac{|PQ|}{|OP||OQ|}\right)^2 = \frac{|OP|^2 + |OQ|^2 - |PQ|^2}{|OP|^2|OQ|^2} = \frac{1}{|OP|^2} + \frac{1}{|OQ|^2} = \frac{1}{1^2} + \frac{1}{2^2} \\
& = \frac{3\cos^2}{12} + \frac{4\sin^2}{12} = \frac{3\cos^2}{12} + \frac{4\sin^2}{12} = \frac{7}{12} \\
& \frac{|PQ|}{|OP||OQ|} = \frac{\sqrt{21}}{6}
\end{aligned}$$

23

$$4 \leq x \leq 10$$

$$x^2 - x - 4 = |x - 1|$$

$$x - 1 \leq x^2 - 2x + 3 \leq 0$$

$$x - 3 \leq x - 1 \leq x - 3$$

$$x - 1 \leq x^2 - 5 \leq 0$$

$$x - \sqrt{5} \leq x - \sqrt{5} \leq x - \sqrt{5}$$

$$f(x) = g(x) = \sqrt{5} \cup 3,$$

$$k = 0 \quad g(x) = g(x) = k|x - 1| + |x - 1| - 2$$

$$k|x - 1| + |x - 1| - 2 = 2k - 2$$

$$g(x) = g(x) = 8 - 2k - 2 = 8 - 2k - 3$$

$$f(a) = f(b) = f(c) = 9 - a^2 = a - b^2 = b - c^2 = c$$

$$a^2 - 1 = b^2 - 1 = c^2 - 1 = a - b - c = 3$$

$$a^2 - 1 = 2a - b^2 - 1 = 2b - c^2 - 1 = 2c$$

$$f(a) = f(b) = f(c) = a - b - c = 3 - k = 3 - 0$$