2020−2021

20201206

8 5 40 .

1. $\{a_n\}$ n S_n $a_1=2$ $S_3=12$ $a_6=$

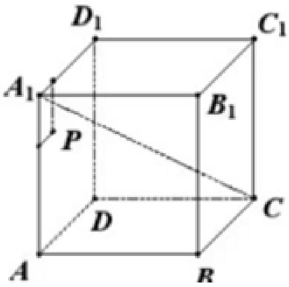
A 8 B 10 C 12 D 14

2. $\{a_n\}$ n $S_n=n^2+1$. a_8

A 65 B 16 C 15 D 14

3. $\{a_n\}$ n S_n $a_3+a_9=a_4+4$ $S_{15}=$

A 45 B 50 C 60 D 80



4. 10 . $ABCD-A_1B_1C_1D_1$ P ADD_1A_1

P A_1D_1 3 P AA_1 2 P 1

Q

A. AA_1B_1B B. BB_1C_1C C. CC_1D_1D D. $ABCD$

5. $\{a_n\}$ $a_1=-9$ $a_3=-1$ $T_n=a_1a_2\cdots a_n(n=1,2,\cdots)$ $\{T_n\}$

A. B.

C. D.

6. $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1\ (a>0,b>0)$ F_1,F_2 $x^2+y^2=a^2+b^2$

AF_2BF_1 p S $p=4\sqrt{2S}$

A $\sqrt{3}$ B $\sqrt{2}$ C $\dfrac{\sqrt{6}}{2}$ D $\dfrac{2\sqrt{3}}{3}$

7. A 1 B 2,3 C 4,5,6 A

7,8,9,10 10000 A 2020

A 5979 B 5980 C 5981 D

8. $\{a_n\}$ $a_1=\dfrac{5}{6}$ $a_{n+1}=\dfrac{(5n+10)a_n}{(n^2+5n+6)a_n+5n+15}$ $a_{99}=$

A $\dfrac{1}{2019}$ B $\dfrac{2018}{2019}$ C $\dfrac{1}{2020}$ D $\dfrac{2019}{2020}$

4 5 20 . .

5 0 3 .

9. $f(x)=\sin\left(2x-\dfrac{}{6}\right)$

A $f(x)$ B $f(x)$ $x=-\dfrac{7}{6}$

C $f(x)$ $\left(-\dfrac{}{4},\dfrac{}{6}\right)$ D $y=f(x)+f\left(x+\dfrac{}{4}\right)$ $-\sqrt{2}$

10.

A k $|PA|-|PB|=|k|$ P

B C A AB AB P

C $2x^2-5x+2=0$

D $\frac{x^2}{25}-\frac{y^2}{9}=1$ $\frac{x^2}{35}+y^2=1$

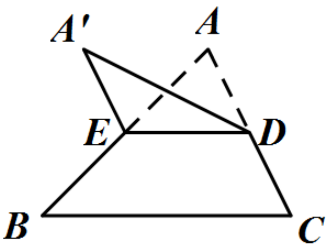
11. $\{a_n\}$ n S_n $a_1>0$ $2a_5+a_{11}=0$

A $a_8<0$ B $n=7$ S_n

C $S_4=S_9$ D $S_n>0$ n 12

12. 2 $\triangle ABC$ D E AC AB $DE//BC$ $\frac{AD}{AC}=\lambda$（$\lambda\in(0,1)$）

$\triangle ADE$ DE $\triangle A'DE$



A $A'E$ F $BF//$ $A'CD$

B $\lambda\in\left(0,\frac{1}{2}\right)$ $A'BC\perp$ $BCDE$

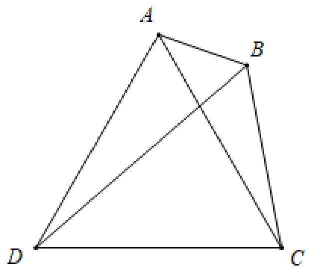
C $\lambda=\frac{1}{2}$ $A'-DE-B$ $|A'B|=\frac{\sqrt{7}}{2}$

D $A'-BCDE$ $f(\lambda)$ $f(\lambda)$ $\frac{2\sqrt{3}}{9}$

4 5 20 2 3 .

13. $\{a_n\}$ n S_n $S_8=18$ $S_{16}=42$ $S_{32}=$______.

14. $\{2n-1\}$ $\{3n-2\}$ $\{a_n\}$ $\{a_n\}$ n ________



15. $ABCD$ $AC=AD=CD=1$ $\angle ABC=120°$

$\sin\angle BAC=\frac{5\sqrt{3}}{14}$ $BC=$______ $BD=$______

16. F $C:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1(a>0,b>0)$ A C B C BF x .

AB 3 C _____________.

6 70 17 10

12 .

17. 10 a_1, d a_1 d $\{a_n\}$ n S_n

$S_5S_6+15=0$

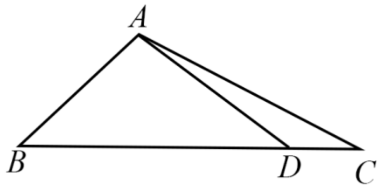
$S_5=5$ S_6 a_1

d

18. 12 $\triangle ABC$ A,B,C $a=3$ $c=\sqrt{2}$ $B=45^\circ$

$\sin C$

BC D $\cos\angle ADC=-\frac{4}{5}$ $\tan\angle DAC$



19. 12 $\{a_n\}$ n S_n $a_1=1$ $a_n\neq 0$ $a_na_{n+1}=\lambda S_n-1$ λ

( ) $a_{n+2}-a_n=\lambda$

λ $\{a_n\}$

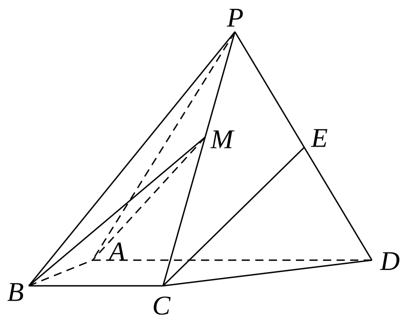
20. 12 $P-ABCD$ PAD $ABCD$

$AB=BC=\frac{1}{2}AD$ $\angle BAD=\angle ABC=90^\circ$ E PD

CE PAB

M PC BM $ABCD$ 45°

$M-AB-D$.



21. 12 $\{a_n\}$ n S_n $a_n+\frac{1}{2}=\sqrt{2S_n+\frac{1}{4}}$ $n\in\mathbf{N}^*$.

$\{a_n\}$

$b_n=\frac{a_{2n+1}}{a_{2n-1}}+\frac{a_{2n-1}}{a_{2n+1}}(n\in N^*)$ $\{b_n\}$ n T_n. n $T_n\ge 2n+m$

m .

22. 12 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$ $A(0,-3)$ F

$|OA|=|OF|$ O

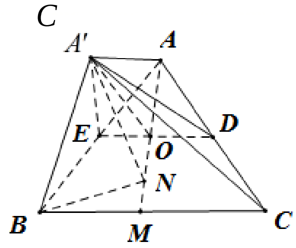
C $3\overrightarrow{OC}=\overrightarrow{OF}$ B B AB C

P P AB AB

2020-2021

A' BCDE N $A'N = OA'\sin 60° = \frac{3}{4}$ $ON = OA'\cos 60° = \frac{\sqrt{3}}{4}$

$\therefore MN = \frac{\sqrt{3}}{4}$ $BN = \sqrt{MN^2 + BM^2} = \frac{\sqrt{19}}{4}$ $\therefore |AB'| = \sqrt{BN^2 + A'N^2} = \frac{\sqrt{7}}{2}$ C



D $\because \frac{AO}{AM} = \frac{AD}{AC} = \frac{DE}{BC} = \lambda$ $\therefore DE = 2\lambda$ $OA' = OA = \sqrt{3}\lambda$

$\therefore S_{BCDE} = \frac{1}{2} \times 2 \times \sqrt{3} - \frac{1}{2} \times 2\lambda \times \sqrt{3}\lambda = \sqrt{3}\left(1-\lambda^2\right)$

$A'DE \perp$ BCDE

$f(\lambda) = \frac{1}{3} \times \sqrt{3}(1-\lambda^2) \times \sqrt{3}\lambda = \lambda - \lambda^3$ $f'(\lambda) = 1 - 3\lambda^2$ $f'(\lambda) = 0$ $\lambda = \frac{\sqrt{3}}{3}$

$0 < \lambda < \frac{\sqrt{3}}{3}$ $f'(\lambda) > 0$ $\frac{\sqrt{3}}{3} < \lambda < 1$ $f'(\lambda) < 0$

$\therefore$ $\lambda = \frac{\sqrt{3}}{3}$ $f(\lambda)$ $\sqrt{}$ $\sqrt{}$ D

4 5 20 2 3 .

13. 108 14. $3n^2 - 2n$ 15. $\frac{5}{7}$ $\frac{8}{7}$ 16. 2

6 70 17 10

12

17. 10

$S_6 = \frac{-15}{S_5} = -3$, $a_6 = S_6 - S_5 = -8$ 2

$\begin{cases} 5a_1 + 10d = 5, \\ a_1 + 5d = -8. \end{cases}$ $a_1 = 7$ $S_6 = -3$ $a_1 = 7$ 5

$S_5 S_6 + 15 = 0$, $(5a_1 + 10d)(6a_1 + 15d) + 15 = 0$ 7

$2a_1^2 + 9a_1 d + 10d^2 + 1 = 0$ $(4a_1 + 9d)^2 = d^2 - 8$ $d^2 \geq 8$ 9

d $d \leq -2\sqrt{2}$ $d \geq 2\sqrt{2}$ 10

18. 12

$b^2 = a^2 + c^2 - 2ac\cos B = 9 + 2 - 2 \times 3 \times \sqrt{2} \times \frac{\sqrt{2}}{2} = 5$ $b = \sqrt{5}$. 3

$\frac{c}{\sin C} = \frac{b}{\sin B} \Rightarrow \sin C = \frac{c \sin B}{b} = \frac{\sqrt{5}}{5}$. 6

$\cos\angle ADC = -\frac{4}{5}$ $\angle ADC \in \left(\frac{\pi}{2}, \pi\right)$ $\sin\angle ADC = \sqrt{1-\cos^2\angle ADC} = \frac{3}{5}$. 7

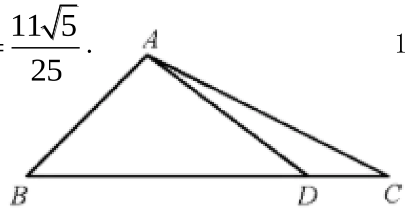
$\angle ADC \in \left(\frac{\pi}{2}, \pi\right)$ $C \in \left(0, \frac{\pi}{2}\right)$ $\cos C = \sqrt{1-\sin^2 C} = \frac{2\sqrt{5}}{5}$ 8

$\sin\angle DAC = \sin\left(\pi - \angle DAC\right) = \sin\left(\angle ADC + \angle C\right)$

$= \sin\angle ADC \cdot \cos C + \cos\angle ADC \cdot \sin C = \frac{3}{5} \times \frac{2\sqrt{5}}{5} + \left(-\frac{4}{5}\right) \times \frac{\sqrt{5}}{5} = \frac{2\sqrt{5}}{25}$. 10

$\angle DAC \in \left(0, \frac{\pi}{2}\right)$ $\cos\angle DAC = \sqrt{1-\sin^2\angle DAC} = \frac{11\sqrt{5}}{25}$. 11

$\tan\angle DAC = \frac{\sin\angle DAC}{\cos\angle DAC} = \frac{2}{11}$. 12



19. 12

$a_n a_{n+1} = \lambda S_n - 1, a_{n+1}a_{n+2} = \lambda S_{n+1} - 1.$ 2

$a_{n+1}(a_{n+2} - a) = \lambda a_{n+1}.$ 4

$a_{n+1} \neq 0$ $a_{n+2} - a_n = \lambda.$ 5

$a_1 = 1$ $a_1 a_2 = \lambda S_1 - 1$ $a_2 = \lambda - 1.$ 6

$a_3 = \lambda + 1.$ $2a_2 = a_1 + a_3$ $\lambda = 4.$ $a_{n+2} - a_n = 4$ 8

$\{a_{2n-1}\}$ 1 4 $a_{2n-1} = 4n - 3$

$\{a_{2n}\}$ 3 4 $a_{2n} = 4n - 1.$ 10

$a_n = 2n - 1$ $a_{n-1} - a_n = 2$.

$\lambda = 4$ $\{a_n\}$ 12

20. 12

PA F EF BF E PD EF AD $EF = \frac{1}{2}AD$ 1

$\angle BAD = \angle ABC = 90^\circ$ BC AD

$BC = \frac{1}{2}AD$ $EF \underline{=} BC$ $BCEF$ CE BF 3

$BF \subset$ PAB $CE \not\subset$ PAB CE PAB 5

$BA \perp AD$ A $\overrightarrow{AB}$ x $|\overrightarrow{AB}|$

$A - xyz$ $B(1,0,0)$ $C(1,1,0)$ $P(0,1,\sqrt{3})$ 6

$\overrightarrow{PC} = (1,0,-\sqrt{3})$ $\overrightarrow{AB} = (1,0,0)$

$M(x,y,z)$ $(0 < x < 1)$ $\overrightarrow{BM} = (x-1,y,z)$ $\overrightarrow{PM} = (x, y-1, z-\sqrt{3})$

BM $ABCD$ 45°

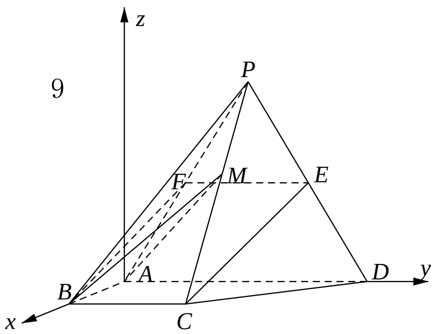
$\vec{n} = (0,0,1)$ $ABCD$

$|\cos < \overrightarrow{BM}, \vec{n} >| = \sin 45^\circ$ $\frac{|z|}{\sqrt{(x-1)^2 + y^2 + z^2}} = \frac{\sqrt{2}}{2}$

$(x-1)^2 + y^2 - z^2 = 0$ 7

M PC $\overrightarrow{PM} = \lambda\overrightarrow{PC}$ $x = \lambda$ $y = 1$ $z = \sqrt{3} - \sqrt{3}\lambda$ 8

$$\begin{cases} x = 1 + \frac{\sqrt{2}}{2} \\ y = 1 \\ z = -\frac{\sqrt{6}}{2} \end{cases} \qquad \begin{cases} x = 1 - \frac{\sqrt{2}}{2} \\ y = 1 \\ z = \frac{\sqrt{6}}{2} \end{cases}$$ 9



$(1$ $\frac{\sqrt{2}}{2}, 1, \frac{\sqrt{6}}{2})$ $\overrightarrow{AM} = (1 - \frac{\sqrt{2}}{2}, 1, \frac{\sqrt{6}}{2})$

$\vec{m} = (x_0, y_0, z_0)$ ABM

$$\begin{cases} \vec{m} \cdot \overrightarrow{AM} = 0 \\ \vec{m} \cdot \overrightarrow{AB} = 0 \end{cases} \qquad \begin{cases} (2-\sqrt{2})x_0 + 2y_0 + \sqrt{6}z_0 = 0 \\ x_0 = 0 \end{cases}$$ $\vec{m} = (0, -\sqrt{6}, 2)$ 11

$\cos < \vec{m}, \vec{n} > = \frac{\vec{m} \cdot \vec{n}}{|\vec{m}||\vec{n}|} = \frac{\sqrt{10}}{5}$

$M - AB - D$ $\frac{\sqrt{10}}{5}$ 12

21. 12

$a_n+\frac{1}{2}=\sqrt{2S_n+\frac{1}{4}}$ $2S_n=a_n^2+a_n$ 1

$n=1$ $a_1=1$. 2

$n\geq 2$ $2S_{n-1}=a_{n-1}^2+a_n-a_{n-1}$

– $2a_n=a_n^2-a_{n-1}^2+a_n-a_{n-1}$ $(a_n+a_{n-1})(a_n-a_{n-1}-1)=0$ 4

$a_n>0$ $a_n-a_{n-1}=1(n\geq 2)$

$\{a_n\}$ 1 1 $a_n=n$. 6

$b_n=\frac{2n+1}{2n-1}+\frac{2n-1}{2n+1}=2+\frac{2}{2n-1}-\frac{2}{2n+1}=2+2(\frac{1}{2n-1}-\frac{1}{2n+1})$ 8

$T_n=2n+2(1-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\cdots+\frac{1}{2n-1}-\frac{1}{2n+1})=2n+2-\frac{2}{2n+1}$. 10

$2n+2-\frac{2}{2n+1}\geq 2n+m$ n $m\leq\frac{4}{3}$. 12

22. 12

$\because$ $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$ $A(0,-3)$ $\therefore b=3$ 1

$|OA|=|OF|$ $c=b=3$ $a^2=b^2+c^2$ $a^2=3^2+3^2=18$ 3

$\frac{x^2}{18}+\frac{y^2}{9}=1$ 4

$\because$ AB C P $CP\perp AB$ 5

AB CP

AB k AB $y+3=kx$ $y=kx-3$

$\begin{cases} y=kx-3 \\ \frac{x^2}{18}+\frac{y^2}{9}=1 \end{cases}$ y $(2k^2+1)x^2-12kx=0$ $x=0$ $x=\frac{12k}{2k^2+1}$.

$x=\frac{12k}{2k^2+1}$ $y=kx-3$ $y=k\cdot\frac{12k}{2k^2\quad 1}-3=\frac{6k^2-3}{2k^2\quad 1}$

B $\left(\frac{12k}{2k^2+1},\frac{6k^2-3}{2k^2+1}\right)$ 7

P AB A $(0,-3)$ P $\left(\frac{6k}{2k^2+1},\frac{-3}{2k^2+1}\right)$ 8

$3\overrightarrow{OC}=\overrightarrow{OF}$ C $(1,0)$ 9

CP $k_{CP}=\frac{\frac{-3}{2k^2+1}-0}{\frac{6k}{2k^2+1}-1}=\frac{3}{2k^2-6k+1}$ 10

$CP\perp AB$ $k\cdot\frac{3}{2k^2-6k+1}=-1$ $2k^2-3k+1=0$

$k=\frac{1}{2}$ $k=1$. 11

AB $y=\frac{1}{2}x-3$ $y=x-3$. 12