

2020-2021

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1. $C_1: x^2 + y^2 - 4x - 4y + 4 = 0$ $C_2: x^2 + y^2 - 4x + 4y - 12 = 0$.

10. $x^2 - y^2 - 4x + 1 = 0$

A $(2, 0)$

B $(y, 0)$

C $x - 3y - 2 = 0$

D $x - y - 2 = 0$

11. $ABCD \cong A_1B_1C_1D_1$

A $ABC_1D_1 \quad |\overline{AB}| \parallel |\overline{BC_1}| \quad B \quad \overline{AD_1} \quad \overline{A_1B}$

C $(\overline{AA_1} \cdot \overline{A_1D_1} \cdot \overline{A_1B_1})^2 = 3 \overline{A_1B_1}^2 \quad D \quad \overline{A_1C} \cdot (\overline{A_1B_1} \cdot \overline{A_1D_1}) = 0$

12. $ABCD \cong A_1B_1C_1D_1 \quad 1 \quad E, F, G \quad BC, CC_1, BB_1$

A $D_1D \quad AF$

B $A_1G \quad AEF$

C $AEF \quad \frac{9}{8}$

D $C \quad G \quad AEF$

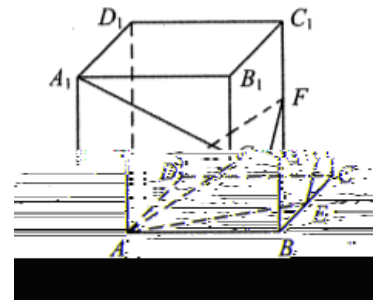
4

5

20

2

3



13. $O \quad ABC \quad D \quad BC \quad P \quad AD$

$\overline{OP} = x\overline{OA} + y\overline{OB} + z\overline{OC} \quad x = y = z = \frac{1}{3}$

14. $C: x^2 + y^2 - 2ax - 12y + 2a^2 = 0 \quad C$

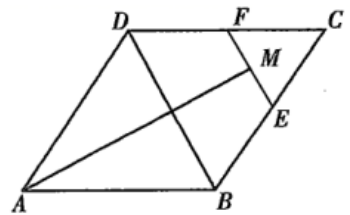
$C \quad l: mx + ny - 6 = 0 \quad m > 0 \quad n > 0$

$\frac{mn}{3m + n}$

15. $ABCD \quad AB = 2, \quad \angle BAD = 60^\circ$

$EF \quad M \quad \overline{AM} = m\overline{AB} + \frac{2}{3}\overline{AD} \quad (m > 0, R)$

$m = \frac{1}{3} \quad \overline{AM} \cdot \overline{BD} = \frac{2}{3}$



16. $M \quad \frac{x^2}{16} + \frac{y^2}{12} = 1 \quad O$

$C: x^2 + y^2 = 25$

$\overline{OA} \cdot \overline{OB} \cdot \overline{OM} = \vec{0}$

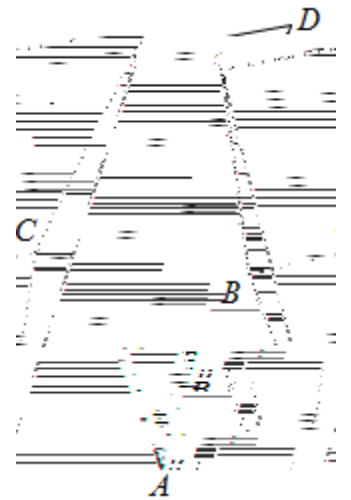
1 $AB \quad N \quad N$

2 $|AB|$

12

17. $10 \quad m \quad l_1 \quad 2x \quad my \quad 1 \quad l_2 \quad mx \quad 8y \quad m \quad 2.$
 $l_1 \quad l_2 \quad m$
 $l_1 \quad l_2 \quad m \quad A \quad A$

18. $12 \quad A \quad BCDE \quad BE \quad ABC$
 $DE // BC \quad DE \quad EB \quad AB \quad 3BC \quad 3 \quad AC \quad \sqrt{10}.$
 $DE \quad ABE$
 $AD \quad M \quad CM \quad AE \quad M$
 $B \quad AD \quad E$



19. $12 \quad \triangle ABC \quad A, B, C \quad \vec{m} \quad (2 \cos \frac{C}{2} \quad \sin C)$
 $\vec{n} = (\cos \frac{C}{2} \quad 2 \sin C) \quad \vec{m} \quad \vec{n}$
 $\frac{C}{2} \quad \frac{1}{2} \quad \sin(A \quad B)$

20. 12 1 $ABCD$ $AB \parallel CD$ AE CD BF CD
 AB AE 2 CD 5 DE 1 $ABCD$ AE BF ADE $ABFE$
 2 .

AF BD

$CF \parallel DE$

AB

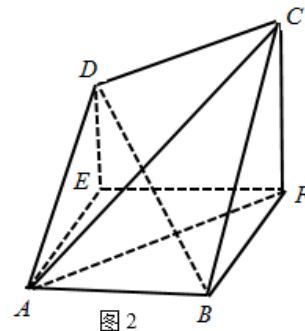
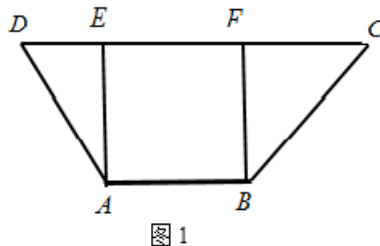
P

CP

ACD

$$\frac{\sqrt{6}}{18}$$

AP



21. 12 ABC A, B, C $(2a - c)\cos B - b\cos C$

B

AB AC

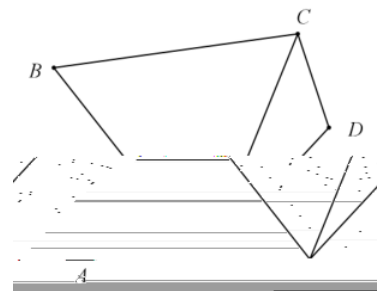
AC

D

AD $2CD$ 4

D

$ABCD$



22. 12 M $C: x^2 - \frac{y^2}{4} = 1$ M y N

P $\overline{NP} = 2\overline{NM}$.

P

E

$A(0, 2)$

$y = kx + \frac{2}{3}$ P

k

AG AH

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8 5 40 .

1 4 AAAC 5 8 BBBA

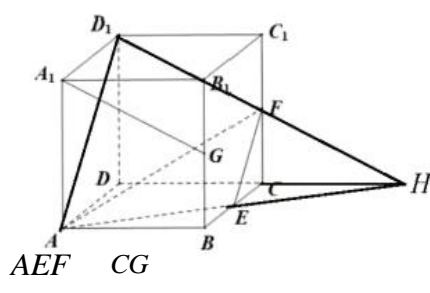
8. $2\sin C \overline{CB} \quad 4\sin A \overline{CA} \quad 3\sin B \overline{AB} \quad 2\sin C \overline{CA} \overline{AB} \quad 4\sin A \overline{CA} \quad 3\sin B \overline{AB},$
 $2\sin C \quad 4\sin A \quad \overline{CA} \quad 3\sin B \quad 2\sin C \quad \overline{AB}, \quad \overline{CA}, \overline{AB} \quad ,$
 $2\sin C \quad 4\sin A \quad 3\sin B \quad 2\sin C \quad 0 \quad 2c \quad 4a \quad 3b \quad 2c \quad 0,$
 $a \quad \frac{1}{2}c, \quad - \quad , \quad c \quad 6, \quad , \quad b \quad 4, \quad \therefore a^2 \quad b^2 \quad c^2 \quad 9 \quad 16 \quad 36 \quad 11 \quad 0$
 $C \quad , \quad ABC \quad ,$
 $4 \quad 5 \quad 20 \quad .$

5 0 3 .

9 ABD 10 ABC 11 ACD 12 BC

12. $DD_1 \quad N \quad AN \quad AN \quad AF \quad ADD_1A_1$
 $AN \quad DD_1 \quad AF \quad DD_1$
 $B_1C_1 \quad M \quad A_1M, GM \quad A_1M // AE, GM // EF$
 $A_1MG // \quad AEF \quad A_1G // \quad AEF$
 $AD_1 \quad D_1F \quad AEFD_1 \quad AEF$

$D_1H \quad AH \quad \sqrt{5} \quad A_1D \quad \sqrt{2}$
 $S_{AD_1H} \quad \frac{1}{2} \sqrt{2} \sqrt{(\sqrt{5})^2 - (\frac{\sqrt{2}}{2})^2} \quad \frac{3}{2}$
 $S_{AEFD_1} \quad \frac{3}{4} S_{AD_1H} \quad \frac{9}{8}$
 $C \quad G \quad AEF$
 $AEF \quad CG \quad CG \quad EF \quad O$
 $O \quad CG$



13 1 $\frac{1}{2}$ 14 1 $\frac{3}{16}$ 15 $\frac{5}{6}$ $\frac{1}{3}$ 16 $\frac{x^2}{4} \quad \frac{y^2}{3} \quad 1 \quad 8, 4\sqrt{6}$

16. $N \quad x, y \quad M \quad x_0, y_0 \quad AB \quad N \quad \overline{ON} \quad \frac{1}{2} \overline{OA} \quad \overline{OB}$
 $\overline{OA} \quad \overline{OB} \quad \overline{OM} \quad \vec{0} \quad \overline{OA} \quad \overline{OB} \quad \overline{OM}$

$\overline{ON} \quad \frac{1}{2} \overline{OM} \quad \begin{matrix} x_0 & 2x \\ y_0 & 2y \end{matrix} \quad M \quad 2x, 2y \quad \frac{x^2}{16} \quad \frac{y^2}{12} \quad 1$

$\frac{2x^2}{16} \quad \frac{2y^2}{12} \quad 1 \quad N \quad \frac{x^2}{4} \quad \frac{y^2}{3} \quad 1$

$|AB|^2 \quad 4 \cdot 5^2 \quad |CN|^2 \quad 100 \quad 4|CN|^2 \quad N \quad 2\cos \quad , \sqrt{3} \sin \quad 0,2 \quad C \quad 1,0$

$|CN|^2 = 2\cos \quad 1^2 \quad \sqrt{3} \sin \quad 2 \quad 4\cos^2 \quad 4\cos \quad 1 \quad 3\sin^2$

$\cos^2 \quad 4\cos \quad 4 \quad \cos \quad 2^2 \quad 0,2$

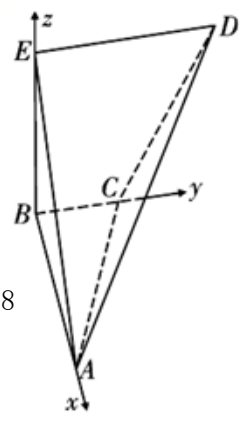
$1 \quad |CN|^2 \quad 9 \quad |AB|^2 \quad 100 \quad 4|CN|^2 \quad 64, 96 \quad |AB| \quad 8, 4\sqrt{6}$

17. 12
10

$$\begin{aligned} \therefore l_1 \quad l_2 \quad & \begin{matrix} m^2 & 16 & 0 \\ 2m & 2 & m \end{matrix} \quad m \quad 4 \quad & \begin{matrix} 5 \\ 2 \end{matrix} \\ 2x \quad my \quad 1 & & x \quad \frac{m-2}{m-4} \quad y \quad \frac{1}{m-4} & 8 \\ mx \quad 8y \quad m-2 & & & \\ A \quad \frac{m-2}{m-4}, \quad \frac{1}{m-4} & & & \\ \therefore x \quad \frac{m-2}{m-4} \quad \frac{m-4-2}{m-4} \quad 1 \quad \frac{2}{m-4} \quad 1 \quad 2y \quad x \quad 2y \quad 1 \quad 0 \quad y \quad 0. & & 10 \\ A \quad x \quad 2y \quad 1 \quad 0. & & & \end{aligned}$$

18. 12

$$\begin{aligned} \therefore DE \quad EB \quad AB \quad 3BC \quad 3 \quad BC \quad 1. \quad \therefore AC \quad \sqrt{10} \\ AB^2 \quad BC^2 \quad AC^2 \quad AB \quad BC & 1 \\ \therefore BE \quad ABC \quad BC \quad ABC \quad BE \quad BC & 2 \\ \therefore AB \quad BE \quad B \quad BC \quad ABE & 3 \\ \therefore DE \parallel BC \quad DE \quad ABE. & 4 \\ B \quad BA, BC, BE \quad x, y, z & \\ D(0,3,3) \quad E(0,0,3) & 5 \\ \overline{CA} \quad (3, 1, 0) \quad \overline{AD} \quad (3, 3, 3) \quad \overline{AE} \quad (3, 0, 3). \\ \overline{AM} \quad \overline{AD} \quad [0,1] \quad \overline{AM} \quad (3, 3, 3) & 7 \\ \overline{CM} \quad \overline{CA} \quad \overline{AD} \quad (3 \quad 3 \quad 3 \quad 1, 3) \\ \overline{AE} \quad \overline{CM} \quad 9 \quad 9 \quad 9 \quad 0 \quad \frac{1}{2} \quad M \quad AD & 8 \\ BAD \quad \vec{n}_1 \quad x_1, y_1, z_1 \quad \therefore \overline{BD} \quad (0, 3, 3) \quad \overline{BA} \quad (3, 0, 0) \\ \vec{n}_1 \quad \overline{BD} \quad 0, \quad 3y_1 \quad 3z_1 \quad 0, \quad y_1 \quad 1 \quad \vec{n}_1 \quad (0, 1, 1). & 9 \\ \vec{n}_1 \quad \overline{BA} \quad 0, \quad 3x_1 \quad 0, \\ ADE \quad \vec{n}_2 \quad x_2, y_2, z_2 \quad \overline{AD} \quad (3, 3, 3) \quad \overline{ED} \quad (0, 3, 0) \\ \vec{n}_2 \quad \overline{AD} \quad 0 \quad 3x_2 \quad 3y_2 \quad 3z_2 \quad 0, \quad x_2 \quad 1 \quad \vec{n}_2 \quad (1, 0, 1) & 10 \\ \vec{n}_2 \quad \overline{ED} \quad 0 \quad 3y_2 \quad 0, \\ \cos \langle \vec{n}_1, \vec{n}_2 \rangle \quad \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} \quad \frac{1}{2}. & 11 \end{aligned}$$



$$B \quad AD \quad E \quad \frac{1}{3} \cdot 12$$

19.

12

$$\vec{m} \cdot \vec{n} = 0 \quad 2\cos^2 \frac{C}{2} - 2\sin^2 C = 0 \quad 1$$

$$1 - \cos C = 2(1 - \cos^2 C) = 0 \quad 3$$

$$2\cos^2 C - \cos C - 1 = 0 \quad \cos C = 1 \quad \cos C = \frac{1}{2} \quad 5$$

$$0 < C < \pi \quad C = \frac{\pi}{3} \quad 6$$

$$a^2 = b^2 + \frac{1}{2}c^2 \quad a^2 = b^2 + \frac{1}{2}c^2$$

$$\sin(A - B) = \sin A \cos B - \sin B \cos A = \frac{a}{2R} \cdot \frac{a^2 - c^2 - b^2}{2ac} - \frac{b}{2R} \cdot \frac{b^2 - c^2 - a^2}{2bc} \quad 10$$

$$= \frac{2a^2 - b^2}{4cR} - \frac{c^2 - c}{4R} = \frac{1}{2} \sin C = \frac{\sqrt{3}}{4} \quad 12$$

20.

12

$$ADE \cap ABFE = DE \quad ADE \cap ABFE = AE \quad DE \cap AE = E$$

$$DE \cap ABFE = AF \quad ABFE \cap DE = E \quad 2$$

$$ABFE \cap AF = BE \quad BE \cap DE = E \quad 3$$

$$BE, DE \cap BDE = AF \quad BDE \cap DE = E \quad 4$$

$$BD \cap BDE = AF \quad BD \cap DE = E \quad 5$$

$$DE \cap EA = EF$$

$$CF \parallel DE \quad CF \cap ABFE = F$$

$$A(2,0,0) \quad B(2,2,0) \quad C(0,2,2) \quad D(0,0,1) \quad 6$$

$$\vec{AD} = (-2,0,1) \quad \vec{AC} = (-2,2,2)$$

$$\vec{ACD} \cdot \vec{n} = (x, y, z)$$

$$\vec{AD} \cdot \vec{n} = 0 \quad 2x - z = 0$$

$$\vec{AC} \cdot \vec{n} = 0 \quad 2x + 2y + 2z = 0$$

$$x = 1 \quad \vec{n} = (1, 1, 2)$$

$$P(2, t, 0) \quad \vec{CP} = (2, t - 2, -2) \quad 9$$

$$CP \perp ACD$$

$$\sin \theta = |\cos \langle \vec{n}, \vec{CP} \rangle| = \frac{|2 + 2(t-2) - 4|}{\sqrt{6} \sqrt{8 + (t-2)^2}} = \frac{\sqrt{6}}{18} \quad t = 1 \quad t = \frac{3}{2} \quad 10$$

$$AP = 1.$$

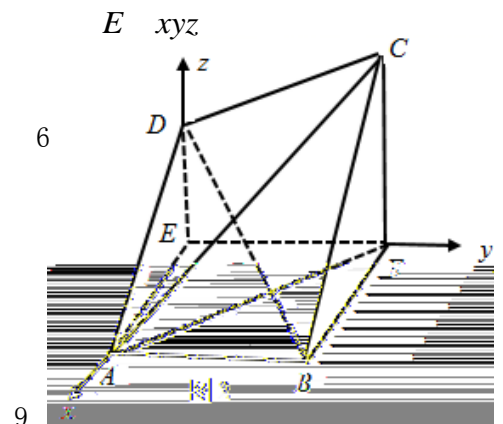
21.

12

$$\sin A \cos B = (2\sin A - \sin C) \cos B = \sin B \cos C \quad 2$$

$$2\sin A \cos B = \sin B \cos C + \sin C \cos B = \sin(B + C)$$

$$2\sin A \cos B = \sin A \quad \therefore \sin A = 0 \quad \cos B = \frac{1}{2} \quad 4$$



$$\because 0 < B < \frac{\pi}{3} \quad B < \frac{\pi}{3} \quad 5$$

$$ABC \quad (2a - c) \frac{a^2 - c^2 - b^2}{2ac} - b \frac{a^2 - b^2 - c^2}{2ab} \quad 2$$

$$a^2 - c^2 - b^2 - ac \cos B = \frac{a^2 - c^2 - b^2}{2ac} = \frac{1}{2} \because 0 < B < \frac{\pi}{3} \quad 4$$

$$B < \frac{\pi}{3} \quad 5$$

$$B < \frac{\pi}{3} \quad AB < AC < BC$$

$$D \quad ABC \quad AC^2 = 16 - 4 + 16\cos \frac{2\pi}{3} = 20 - 16\cos \frac{2\pi}{3} \quad 7$$

$$S_{ABC} = \frac{1}{2} AC^2 \sin \frac{\pi}{3} = 5\sqrt{3} - 4\sqrt{3} \cos \frac{2\pi}{3}, S_{ACD} = \frac{1}{2} \cdot 4 \cdot 2 \sin \frac{\pi}{3} = 4\sin \frac{\pi}{3} \quad 9$$

$$ABCD \quad S = 5\sqrt{3} - 4\sqrt{3} \cos \frac{2\pi}{3} + 4\sin \frac{\pi}{3} = 5\sqrt{3} + 8\sin \left(\frac{\pi}{3} \right) \quad 11$$

$$\because 0 < \frac{\pi}{3} < \frac{\pi}{2} < \frac{2\pi}{3}$$

$$\frac{\pi}{3} < \frac{\pi}{2} < \frac{2\pi}{3} \quad S_{\max} = 8 + 5\sqrt{3}$$

$$D \quad \frac{5}{6} \quad ABCD \quad 8 + 5\sqrt{3} \quad 12$$

22. 12

$$1 \quad P(x, y), \quad M \left(\frac{x}{2}, y \right) \quad 2$$

$$M \quad \frac{x^2}{4} + \frac{y^2}{4} = 1 \quad 4$$

$$P \quad x^2 + y^2 = 4$$

$$G(x_1, y_1), H(x_2, y_2) \quad x_1 = 0, x_2 = 0 \quad k_{AG} = \frac{y_1 - 2}{x_1}, k_{AH} = \frac{y_2 - 2}{x_2} \quad 5$$

$$k_{AG} \cdot k_{AH} = \frac{y_1 - 2}{x_1} \cdot \frac{y_2 - 2}{x_2} = \frac{k^2 x_1 x_2 - \frac{4}{3} k x_1 - x_2 + \frac{16}{9}}{x_1 x_2} \quad 7$$

$$\begin{aligned} x^2 + y^2 = 4 \\ y = kx + \frac{2}{3} \end{aligned} \quad 1 - k^2 x^2 - \frac{4}{3} kx - \frac{32}{9} = 0, \quad \begin{aligned} x_1 + x_2 &= \frac{\frac{4}{3}k}{1 - k^2} \\ x_1 x_2 &= \frac{\frac{32}{9}}{1 - k^2} \end{aligned} \quad 10$$

$$k_{AG} \cdot k_{AH} = k_{AG} \cdot k_{AH} = \frac{k^2 \frac{32}{9} - \frac{4}{3}k \frac{4}{3}k - \frac{16}{9}}{\frac{32}{9} - \frac{16}{9}} = \frac{16}{\frac{32}{9} - \frac{16}{9}} = \frac{1}{2} \quad 12$$