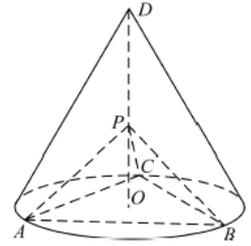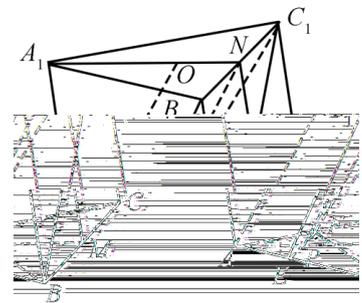## 2020–2021

1 2020 D O ABC

P DO $APC=90$

1 PAB PAC

2 $DO=\sqrt{2}$ $\sqrt{3}$ $P–ABC$.

2 2020 $ABC–A_1B_1C_1$

BB_1C_1C M N BC B_1C_1 P AM

B_1C_1 P AB E AC F

1 $AA_1//MN$ A_1AMN EB_1C_1F

2 O $A_1B_1C_1$ $AO=AB=6$ $AO//$ EB_1C_1F

$MPN=\dfrac{}{3}$ $B–EB_1C_1F$

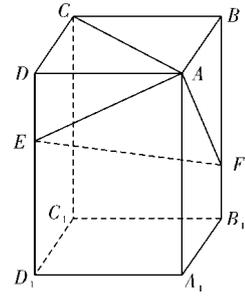3 2020 $ABCD - A_1B_1C_1D_1$ E F DD_1 BB_1

$2DE = ED_1$ $BF = 2FB_1$

1 $AB = BC$ $EF \perp AC$ 2 C_1 AEF

4 2020 ABC $A_1B_1C_1$ AB AC B_1C ABC

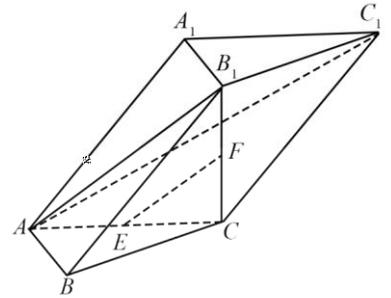E F AC B_1C 1 EF AB_1C_1 2

AB_1C ABB_1

5 2020 *ABC* *DEF* *ACFD* *ABC*

$ACB= ACD=45$ $DC =2BC$ *EF* *DB* *DF*

DBC
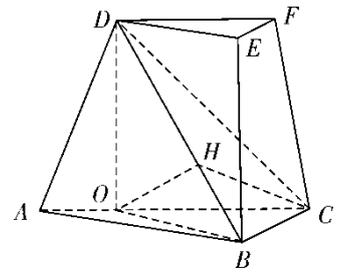
6 2019 *ABCD* $A_1B_1C_1D_1$ $AA_1=4$ $AB=2$ $BAD=60°$

E *M* *N* *BC* BB_1 A_1D .

1 *MN* C_1DE 2 *C* C_1DE
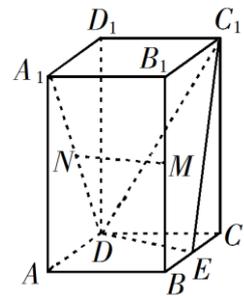
7 2019 $ABCD–A_1B_1C_1D_1$ $ABCD$ E AA_1 BE EC_1

1 BE EB_1C_1

2 $AE=A_1E$ $AB=3$ $E-BB_1C_1C$

8 2019 1 $ADEB$ Rt ABC $BFGC$ $AB=1$

$BE=BF=2$

$FBC=60°$ AB BC BE BF DG 2

1 2 A C G D ABC $BCGE$

2 2 $ACGD$.

1 2020 ... *D* ... *O* ... *ABC*

P *DO* ... *APC*=90

1 ... *PAB* ... *PAC*

2 *DO*=$\sqrt{2}$ ... $\sqrt{3}$ ... *P–ABC* .

1 ... *PA=PB= PC*

ABC ... *PAC* *PAB* *PAC* *PBC*

APC =90 ° *APB*=90 ° *BPC*=90 °

PB *PA* *PB* *PC* *PB* *PAC* *PAB* *PAC*

2 ... *r* ... *l* ... $rl=\sqrt{3}$ $l^2 - r^2 = 2$ ... *r*=1 $l=\sqrt{3}$

$AB = \sqrt{3}$ 1 $PA^2 + PB^2 = AB^2$ $PA = PB = PC = \dfrac{\sqrt{6}}{2}$

P-ABC $\dfrac{1}{3} \times \dfrac{1}{2} \times PA \times PB \times PC = \dfrac{1}{3} \times \dfrac{1}{2} \times (\dfrac{\sqrt{6}}{2})^3 = \dfrac{\sqrt{6}}{8}$

2 2020 ... *ABC*–*A₁B₁C₁* ... *BB₁C₁C* ... *M*

N *BC* *B₁C₁* ... *P* *AM* ... *B₁C₁* *P* ... *AB* *E* *AC* *F*

1 *AA₁//MN* ... *A AMN* ... *EB₁C₁F* 2 *O* *A₁B₁C₁* ... *AO=AB*=6 *AO//*

EB₁C₁F *MPN*=$\dfrac{}{3}$ *B–*

5

1

3   2020 $ABCD-A_1B_1C_1D_1$ E F

DD_1   BB_1 $2DE=ED_1$ $BF=2FB_1$

1      $AB=BC$ $EF\perp AC$ 2 C_1 AEF

1   BD B_1D_1 $AB=BC$ $ABCD$

$AC\perp BD$ $BB_1\perp$ $ABCD$ $AC\perp BB_1$

$AC\perp$ BB_1D_1D $EF\subset$ BB_1D_1D $EF\perp AC$

2   AA_1 G $AG=2GA_1$ GD_1 FC_1 FG

$D_1E=\dfrac{2}{3}DD_1$ $AG=\dfrac{2}{3}AA_1$ DD_1 $=$ AA_1 ED_1 $=$ AG ED_1GA

AE GD_1 $B_1F=\dfrac{1}{3}BB_1$ $A_1G=\dfrac{1}{3}AA_1$ BB_1 $=$ AA_1 FG $=$ A_1B_1 FG $=$ C_1D_1

FGD_1C_1 GD_1 FC_1 AE FC_1 A,E,F,C_1 C_1 AEF
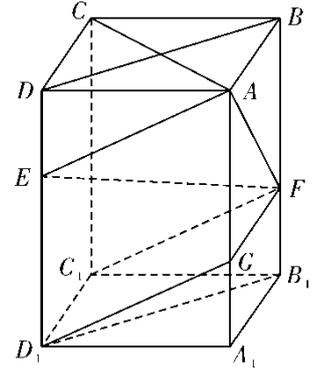
4   2020 ABC $A_1B_1C_1$ AB AC B_1C

ABC E F AC B_1C 1 EF AB_1C_1

2   AB_1C ABB_1

1   AC,B_1C EF AB_1.

$EF\not\subset$ AB_1C_1 $AB_1\subset$ AB_1C_1 EF AB_1C_1.

2        $B_1C\perp$ ABC $AB\subset$ ABC, $B_1C\perp AB$.

$AB\perp AC$,$B_1C\subset$ $AB_1C_1,AC\subset$ $AB_1C,B_1C\bigcap AC=C,$ $AB\perp$ AB_1C. $AB\subset$

ABB_1, $AB_1C\perp$ ABB_1.
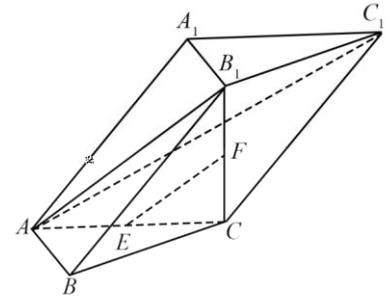
5   2020 ABC DEF $ACFD$ ABC $\angle ACB=\angle ACD=45°$ DC

$=2BC$ EF DB DF DBC

D $DO\perp AC$ AC O OB.

$\angle ACD=45°$ $DO\perp AC$ $CD=\sqrt{2}CO$

$ACFD$ ABC DO ABC $DO\perp BC$.

$\angle ACB=45°$ $BC=\dfrac{1}{2}CD=\dfrac{\sqrt{2}}{2}CO$ $BO\perp BC$.
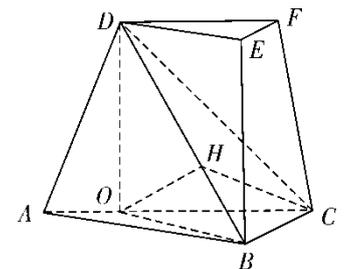
BC   BDO   BC   DB.

ABC – DEF   BC   EF   EF ⊥ DB.

O   OH ⊥ BD   BD   H   CH.

ABC – DEF   DF   CO   DF   DBC   CO   DBC   .

BC ⊥   BDO   OH ⊥ BC   OH ⊥   BCD   ∠OCH   CO   DBC   .

$CD = 2\sqrt{2}$. $DO = OC = 2, BO = BC = \sqrt{2}$ $BD = \sqrt{6}, OH = \dfrac{2}{3}\sqrt{3}$

$\sin \angle OCH = \dfrac{OH}{OC} = \dfrac{\sqrt{3}}{3}$ DF   DBC $\dfrac{\sqrt{3}}{3}$.

6   2019 $ABCD\ A_1B_1C_1D_1$ $AA_1 = 4$

$AB = 2$ $BAD = 60°$ E   M   N   BC BB_1 A_1D .

1   MN C_1DE 2   C C_1DE

        1   B_1C, ME .   M   E BB_1, BC

ME B_1C $ME = \dfrac{1}{2}B_1C$.   N A_1D $ND = \dfrac{1}{2}A_1D$.

        $A_1B_1 =\ DC$ $B_1C =\ A_1D$ $ME =\ ND$

        MNDE   MN   ED . $MN \not\subset$ C_1DE MN C_1DE .

        2   C C_1E H. $DE \perp BC$ $DE \perp C_1C$

DE C_1CE DE   CH.   CH C_1DE CH   C C_1DE

        CE = 1 $C_1C = 4$ $C_1E = \sqrt{17}$ $CH = \dfrac{4\sqrt{17}}{17}$.   C

C_1DE $\dfrac{4\sqrt{17}}{17}$.

7   2019 $ABCD - A_1B_1C_1D_1$ ABCD

        E AA_1 BE EC_1

    1   BE EB_1C_1

    2        $AE = A_1E$ $AB = 3$ $E - BB_1C_1C$

        1   B_1C_1 ABB_1A_1 $BE \subset$ ABB_1A_1
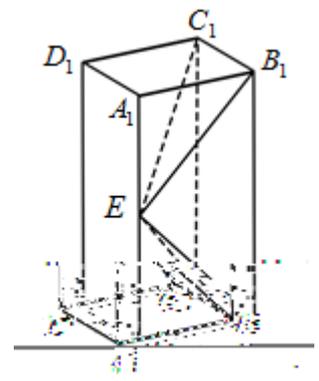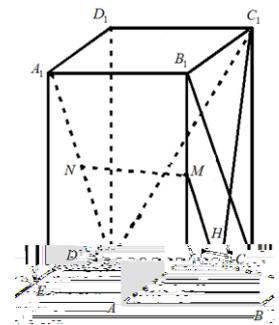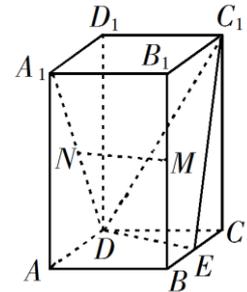
$B_1C_1 \perp BE$ $BE \perp EC_1$ BE EB_1C_1

2   1 $BEB_1 = 90°$.

Rt$\triangle ABE$ Rt$\triangle A_1B_1E$ $\angle AEB = \angle A_1EB_1 = 45°$

$AE=AB=3$ $AA_1 = 2AE = 6$. $EF \perp BB_1$ F EF BB_1C_1C $EF = AB = 3$

$E - BB_1C_1C$ $V = \dfrac{1}{3} \times 3 \times 6 \times 3 = 18$

8   2019   1 $ADEB$ Rt ABC $BFGC$ $AB=1$

$BE=BF=2$

$FBC=60°$ AB BC BE BF DG 2

1   2 A C G D ABC $BCGE$

2   2 $ACGD$.

1     $AD /\!/ BE$ $CG /\!/ BE$ $AD /\!/ CG$ AD CG A C G

D $AB \perp BE$ $AB \perp BC$ $AB \perp$ $BCGE$

$AB \subset$ ABC $ABC \perp$ $BCGE$

2   CG M EM DM. AB DE $AB \perp$ $BCGE$ $DE \perp$
$BCGE$ $DE \perp CG$.

$BCGE$ $EBC=60°$ $EM \perp CG$ $CG \perp$ DEM

$DM \perp CG$ Rt DEM $DE=1$ $EM=\sqrt{3}$ $DM=2$ $ACGD$ 4

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